A wavelet based method for estimating the damping ratio in Stanamic pile load tests

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ABSTRACT

A wavelet based method is proposed to evaluate the time-dependent damping ratio in statnamic load tests by the continuous wavelet transform and half-power bandwidth method. The displacement along the pile during a statnamic test is described by a linear shape function, although the pile is assumed to be a single degree of freedom system (SDOF). The damping ratio is calculated by the half-power bandwidth method from the time-frequency spectra of continuous wavelet transform for the statnamic pile load test. A numerical simulation and two field statnamic tests were analyzed to verify the applicability of the proposed method, and the outcomes were compared with the results obtained using the unloading point method (UPM) and a method in literature. The damping ratio obtained with the proposed method is satisfactory and provides an additional interpretation measure for statnamic load tests.

Key Words: Statnamic load test, damping coefficient, continuous wavelet transform.

1. Introduction

Piles are used to transmit surface loadings to lower levels in the soil strata. Such foundations are most often used with heavy structures or when the shallow soils are weak. The load transfer mechanism from a pile to the soil is complicated. In most large projects, static pile load tests have to be performed to acquire pile design data. Although this is a reliable method to determine the design load of a pile, the static load test is expensive and time consuming. Therefore, the statnamic load test is used for economic and faster results. Because the load duration of the statnamic load test is longer than the natural period of the pile, the stress field and the displacement field of the statnamic load test have little relationship with wave propagation (Stokes et al., 2008). The stress and displacement fields are close to those of the conventional static load test, although it actually uses a dynamic load. The velocity and acceleration time history of a pile head can be derived from the displacement time history of the pile head. The test is between the static load test and dynamic load test, and it is called the “statnamic load test” (Stokes et al., 2008; Mullins et al., 2002). Compared to the static load test, the statnamic test is a quicker and more efficient method for the pile load test. For a statnamic test, fuel is burned in the statnamic device; this will produce a high-pressure force and propels a reaction mass upwards. The reaction force will acts in the opposite direction to the pile head. The time histories of the loading and vertical pile head displacement are simultaneously recorded. The dynamic load-displacement curve then can be plotted for estimating the bearing capacity of the statnamic pile. The magnitude of load is directly related to the quantity of fuel used (Lin et al., 2004; Mullins et al., 2002).

To obtain the equivalent static loading versus displacement curve from statnamic load test data, it is necessary to remove components of inertia and damping forces associated with the pile and the surrounding soil. A number of researchers have explored methods of interpreting test data in order to
obtain the static resistance and stiffness of the pile from statnamic test data. The first published approach for interpreting the statnamic test was proposed by Middendorp et al. (1992). They proposed the unloading point method (UPM), which is based on the assumption that a pile is a rigid body, or has a single degree of freedom, thus excluding stress wave behavior from the analysis. This method is the most common method to evaluating the static pile loading response from statnamic test data. In the UPM, it is assumed that the static soil resistance is constant during the time of maximum statnamic loading to the time of maximum displacement. In the state of maximum displacement, the velocity and the resulting damping force equal zero, and the static loading capacity can be calculated by the statnamic force minus the inertial force. Thus, the damping force and the corresponding damping coefficient can be determined from the maximum statnamic force minus the static capacity and the inertial force. This inertial force is calculated at the time when the maximum statnamic force occurs.

The damping coefficient is assumed constant throughout the statnamic test. However, long piles will not meet the UPM criteria, i.e., the rigid body assumption is not appropriate for the long-pile case. Mullins et al. (2002) developed an analysis method that discretizes the pile into smaller segments, termed the segmental unloading point method (SUPM). This method requires that strain gauges have to be installed at several levels of the pile. The pile is divided into several segments, defined by gauge locations. Each segment is considered a rigid body. The displacements, velocities, accelerations and forces can be calculated from the strain gauges and the transducers on the pile. Ealy and Justason (2000) have found that the static curve derived by UPM or SUPM does not fit the test data absolutely. They suggested that it is better to have an adaptable interpreting method to use a complicated damping coefficient, because the damping coefficient is not a constant throughout the statnamic test process. Lin et al. (2004) proposed the structural damping concept for estimating the static pile response from statnamic test data. In this interpretation method, displacement-related damping is used to replace velocity dependent damping. The damping constant is then related with the area enclosed by the load-displacement curve obtained and the maximum displacement of the statnamic test. These previous studies adopt a key assumption: that the damping coefficient is constant throughout the test. In fact, the damping coefficient is not constant during the statnamic test process. Stokes et al. (2008) introduced the instantaneous volume change dependent damping to interpret statnamic test data. They monitored the incremental loss of soil volume associated with the depressed surface for calculating the displacement dependent volume change, for which a number of displacement transducers need to be installed on the surrounding ground surface.

Energy loss and damping in soil can result from several mechanisms, including friction, heat generation, and plastic yielding. However, the damping mechanism is not sufficiently understood, and the damping cannot thus be explicitly analyzed. Viscous damping is commonly used to represent the energy dissipation of the system. Viscous damping can be determined by the half-power bandwidth method (Clough and Penzien, 1993). In the half-power bandwidth method, soil damping is calculated by measuring the frequencies at $1/\sqrt{2}$ of the maximum amplitude and the resonant frequency from a frequency response spectrum. This study adopted the half-power bandwidth method to determine the
damping coefficient and evaluate the equivalent static response of pile. When the half-power bandwidth method is used, the damping is normally expressed as viscous damping, and then the viscous damping force is related to velocity. However, the damping is related to pile-head displacement indicated by the field test data in this study, because the calculated damping ratio is higher during large displacement. Therefore, there exists hysteretic damping effect also. Therefore, we use the term “equivalent viscous” instead of “viscous” in this study to reveal that the damping effect consists of both viscous and hysteretic damping.

Due to the time dependent damping, the time-frequency characteristics of the statnamic test data can be obtained by the continuous wavelet transform. Therefore, the continuous wavelet transforms were performed in this study to obtain the time history of the frequency spectra during the statnamic test process. The soil static resistance could be determined directly from the measured statnamic load-displacement curve after the time history of equivalent viscous damping coefficient is calculated. A numerical simulation and two case studies of field statnamic tests are presented in this paper to confirm the applicability of the proposed method. The results calculated using the UPM and from the analysis method by Lin et al. (2004) are also compared.

2. Method

In this study, we used the Morlet wavelet function and continuous wavelet transform to transform the time history of load and response signals of the statnamic pile load test into a dilation parameter versus time for the loading and response signals. From these transformed signals, we obtained the spectra of the frequency response function versus time of the statnamic pile load test. These spectra were then used to evaluate the time dependent damping ratio by the half-power bandwidth method. The processes are provided below.

2.1 Interpretation of statnamic data

Displacement of the pile head after loading is always larger than the tip. For fixed-end pile, the tip displacement is almost zero. Therefore, we assume the displacement along a pile during the statnamic test is continuous throughout the pile and can be described by a linear shape function from the pile head to tip (Fig. 1). The particle motion at any point can be represented by the linear shape function. Although the pile is assumed to be a single degree of freedom system (SDOF), the displacements along the pile actually vary and are represented in terms of the displacement at the pile head (Fig. 1). Using a linear shape function for displacement is an improvement over the UPM method, and it is a better representation of the fixed-end or relatively long pile situation. If the displacement of the pile head is denoted as a generalized coordinate, \( u(t) \), the displacement at distance \( x \) under the pile head is

\[
\left(1 - \frac{x}{L}\right)u(t), \text{ where } L \text{ is the pile length. The generalized mass, } m^*, \text{ for a pile can be evaluated by Eq. (1) (Clough and Penzien, 1993):}
\]
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\[ m^* = \int_0^L \rho \left(1 - \frac{x}{L}\right)^2 \, dx = \frac{\rho L}{3} = \frac{m}{3} \]

where \( \rho \) is mass per length, and \( m \) is mass of the pile. In Fig. 1, the spring represents the generalized stiffness, the dashpot corresponds to the generalized damping, and the spring and dashpot are assumed to be massless. The generalized stiffness, \( k^* \), includes the distributed stiffness of the pile and the resistance of the surrounding soil. The generalized damping, \( c^* \), is evaluated as:

\[ c^* = \int_0^L c' \left(1 - \frac{x}{L}\right)^2 \, dx = \frac{c'L}{3} = \frac{c}{3} \]

where \( c' \) is the damping coefficient per length, and \( c \) is the total damping coefficient of pile. The pile is then assumed to be a SDOF system when interpreting the static pile load-displacement curve. When a statnamic load, \( F_{\text{stn}}(t) \), is applied to the system, the tendency for motion is resisted by the pile inertia force, \( m^* a(t) \), the soil damping force, \( c^* (t)v(t) \), and the soil static resistance, \( F_u(t) \). The equation of motion describing a statnamic event can be expressed in terms of the dynamic equilibrium of these forces:

\[ m^* a(t) + c^* v(t) + F_u(t) = F_{\text{stn}}(t) \]

The applied statnamic load, \( F_{\text{stn}}(t) \), is measured using the load cell, and the vertical displacement on the pile head, \( u(t) \), can be measured by a laser sensor. The velocity, \( v(t) \), is the rate of change of displacement with respect to time \( t \), and the acceleration, \( a(t) \), is the rate of change of velocity with respect to time \( t \). The static soil resistance can be obtained for

\[ F_u(t) = F_{\text{stn}}(t) - c^* (t)v(t) - m^* a(t) \]

Once the time-dependent damping coefficient, \( c^* (t) \), is determined, the static soil resistance, \( F_u(t) \), can be obtained. Hence, an equivalent static load-displacement curve can be drawn from \( F_u(t) \) and \( u(t) \), which yields the static pile capacity.

### 2.2 The Continuous Wavelet Transform

Conventional Fourier analysis cannot provide information about how the frequency content of a signal changes with time. For a non-stationary signal, whose features vary with time, the frequency spectra are temporally dynamic. To understand the transient changes in the spectra, a time-frequency analysis must be performed. The short-time Fourier transform (STFT) is a conventional time-frequency
signal process method with a “window” moving along the time axis to analyze the signal content in terms of both frequency and time. However, the length of the window in time is constant in STFT, such that it cannot obtain sufficient time resolution at high frequency or a sufficient frequency resolution at low frequency (Addison, 2002). The wavelet transform uses a size-adjustable window, such that the time duration of the window is shorter at high frequency and longer at low frequency. The wavelet transform has high time resolution but low frequency resolution at high frequencies, and, in contrast, it has high frequency resolution but low time resolution at low frequencies (Addison, 2002). The response and loading of the statnamic test are low frequency. For a low frequency signal, it is necessary to use a good frequency resolution function to analyze the signal, and a wavelet function satisfies this requirement. Using a wavelet function to transform a statnamic test signal is very appropriate to obtain sufficient frequency resolution and avoid frequency leakage in the low frequency range.

The wavelet transform is a method of converting a signal into a form where the characteristics of the original signal are more suitable for study. We followed the method explained in Addison (2002) and described the wavelet transform method in the following.

A wavelet function \( \psi(t) \) is used for performing translation and dilation processes to transform the original signal into another form of dilation parameter and time. The Morlet complex wavelet functions, which have both real and imaginary parts, were used in this study. The phase and amplitude components of the signal are obtained by using Morlet complex wavelet. The Morlet wavelet is defined as:

\[
\psi(t) = \pi^{-0.25} \left( e^{i2\pi f_0 t} e^{-\frac{\left(2\pi f_0 t\right)^2}{2}} \right) e^{-t^2/2}
\]  

(5)

where \( f_0 \) is center frequency of the wavelet, and \( i \) denotes \( \sqrt{-1} \). The center frequency is simply the standard deviation of the energy spectrum of the wavelet, and it indicates the range and characteristics of the frequencies that make up the wavelet. Because frequencies are inversely proportional to the dilations, the relationship between frequency and the dilation is defined as (Misiti et al., 2010):

\[
f = \frac{f_n}{a\Delta_t}
\]  

(6)

where \( f_0 \) is the center frequency of a wavelet, \( a \) is a dilation parameter, and \( \Delta_t \) is the sample period. In this study for statnamic pile test, we used the value of 1.5Hz as the center frequency of the Morlet complex wavelet with the Matlab wavelet toolbox. The center frequency is used to relate the frequency spectra obtained from the Fourier transform to those obtained from the wavelet transform. The wavelet described by Eq. (5) is known as the “mother wavelet” of the Morlet wavelet. Its dilated and translated versions are derived and used in the wavelet transform. The dilation and contraction of the wavelet is governed by the dilation parameter, \( a \). The movement of the wavelet along the time axis is governed by translation parameter \( b \). If the mother wavelet includes the dilation parameter, \( a \), and the translation parameter, \( b \), the shifted, dilated and normalized version of the wavelet can be written as
The continuous wavelet transform of a time signal \( f(t) \) is defined as:

\[
W_t(a, b) = \langle f, \psi_{(a,b)} \rangle = |d|^{-0.5} \int_{-\infty}^{\infty} f(t) \psi^*(\frac{t-b}{a}) dt
\]  

(8)

The asterisk indicates the complex conjugate of the wavelet function. The continuous wavelet transform uses the dilated and shifted version of wavelet \( \psi_{(a,b)} \) to decompose the signal.

2.3 Damping of the pile-soil system

The damping of pile-soil system can be determined by the half-power bandwidth method. In the method, the damping ratio of soil can be obtained by harmonic excitation with various frequencies and by determining the amplitude of the magnification factor at each frequency (Clough and Penzien, 1993).

The half-power bandwidth method is appropriate for the steady-state case, but the loading and response of the statnamic test is a transient case. However, a transient load can be decomposed to a series of harmonic loads with respect to frequency by Fourier transformation. Each harmonic loading, in which the individual amplitude is different for that of other frequencies, can produce a steady-state response. Thus, the response due to the transient load can be found by the sum of these corresponding harmonic responses. In order to make the amplitude of the load the same at every harmonic frequency, the “frequency response function” spectra were used. The frequency response function is defined as the transient displacement divided by the transient load, rather than the transient displacement, to obtain the response spectra. Thus, the damping ratio with respect to the loading time history based on the half-power bandwidth method can be calculated from this frequency response function spectra at the corresponding time. In this study, we determined the frequency response function of the statnamic pile load test with the half-power bandwidth method to evaluate the damping ratio of the system. After the damping ratio versus time is obtained, the static soil resistance is evaluated using Eq. (4).
3. Numerical Simulation

3.1 The numerical model for statnamic test

A numerical simulation of statnamic pile test is performed using the finite difference FLAC code (Itasca, 2008). The concrete pile is 20 m long with 1 m diameter inserted into a two layer soil strata.

The axisymmetric numerical model for the statnamic load test simulation is shown in Fig. 2. The upper soil layer is 20 m thick. The soil strata are assumed as Mohr-Coulomb materials.

The material properties of the pile and soil strata are listed in Table 1 and 2. The pile is installed in-place before the initial static equilibrium by assigning the concrete properties in Table 2 to the pile elements. We did not use interface model between the pile and the soil, because the interface parameters have to be assumed and could involve more uncertainties. The initial stress in the ground was obtained by “stepping to equilibrium” approach by observing the maximum unbalanced force reduced to an acceptable small value. The detailed processes for reaching initial equilibrium can be consulted in the (Itasca, 2008). The displacements are initialized to zero after the initial equilibrium stage is reached and before the dynamic loading stage.

The loading and the displacement curves of the simulated statnamic load test is shown in Fig. 3. Fig. 4 demonstrates the results of continuous wavelet transform to analyze the frequency response function of the numerical simulation. Also, there are only one or two larger undulations at different timing as shown in Fig. 4.

The frequency response function in Fig. 4(c) shows single “peak” or undulation at most of the timings during 0.02 ~ 0.1 sec, i.e., the major statnamic loading duration. This result also validates the half-power bandwidth method used for obtaining damping ratio. In other word, the quality of the damping ratio obtained would be better if there is only one “peak” in the plot of wavelet coefficient versus dilation parameter. The quality of damping ratio obtained from the numerical simulation will be better than those of in-situ statnamic loading test because of background noises and heterogeneity in natural geological strata.

The damping ratios are nearly constant except at the beginning and ending of the test, as illustrated in Fig. 5. This could be that the Mohr-Coulomb soil model is insufficient to model the plastic and dynamic behavior of the soils, resulting nearly constant damping.

The simulated static bearing capacity of the pile is estimated in the range of 80 MN as displacement of 10 % of pile diameter to 112 MN at yielding as shown in Fig. 6. Comparison of the static load-displacement curves of the results from the proposed method, UPM, and simulated static load test are depicted in Fig. 7. The static loading curve fits quiet well the predicted curves obtained from this study and this also validates the proposed method.

In Fig. 7, the loop of the load-displacement curve of the proposed method is smaller than that of the UPM. In addition, the residual strain is very small by the proposed method; while the curve of the
UPM shows about 5 mm residual strain after the test. Again, the bearing capacity of the UPM is larger than that from the proposed method. This is due to that the estimation of mass and damping in the proposed method are different from the UPM.

3.2 Wave propagation effect

The vertical stress variation along the pile during the 0.12 sec statnamic loading can be observed in Fig. 8. The vertical stresses are plotted for each time increment of 0.01 sec; therefore, there are totally 12 curves of stresses versus depth. The timing of the stresses in pile close to their maximums is around t=0.06 sec. The stresses are estimated as 77.3 ~ 84.1 MPa. The authors found that there is only one major vertical stress cycle that the pile experienced, from low stress (t=0.01 sec) to high stress (t=0.06 sec) and then back to low stress (t=0.12 sec). This stress cycle is dominated by the statnamic loading.

The stress wave propagation velocity in the simulated concrete pile is about 3000 m/s. During the loading duration of about 0.1 sec, the stress wave will propagates back and forth in the 20-meter simulated pile about 7.5 times. This will result in some inertia resistance in the pile. However, by observing the vertical stress versus time plot for various pile depths (Fig. 9), stress wave propagation effect is not obvious because the stress wave in pile seem to be “buried” in the dominating stress controlled by the statnamic loading.

Therefore, the authors conclude that the statnamic loading dominates the pile behavior and the effect of wave propagation is not significant in this study.

3.3 Displacement shape function and bearing stratum properties

A parametrical study for the cohesion parameter, c, of the bearing stratum (as the layer 2 in Fig. 2) was performed. The vertical displacement profiles along the pile normalized with the pile-head displacement at t=0.06 sec were presented corresponding to the studied cohesion values in Fig. 10.

For the cases of cohesion parameters of 1300, 2200, and 3400 kPa, the shape of the displacement profiles resembles a triangle. In addition, the stronger the cohesion parameter, the shape of the displacement profile will develop closer to a triangle. The also prove that the linear displacement shape function assumption as a triangle (Fig. 1) is reasonable for these cases and an higher order displacement function would not be necessary because of the almost linear displacement profiles.

For the medium strength bearing layers, such as cohesion parameters of 400 and 700 kPa (Fig. 10), the displacement profiles resemble trapezoids. Therefore, a trapezoid shape function would better be assumed for medium strength stratum. For practical application, if a trapezoid shape function is to be presumed, the denominators in Eq. (1) and (2) should be adjusted accordingly; for example, to use a value of three to no less than unity. The estimation of the values of the denominators in Eq. (1) and (2) should refer to the site investigation of the bearing stratum for its strength. For the case of cohesion 400 kPa (Fig. 10), the denominators in Eq. (1) and (2) can be estimated by integrating the Eq. (1) and (2)
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for the trapezoid shape function.

Suppose there is no or weak bearing stratum beneath a pile, the displacement profile will become a "rectangle" and resembles a rigid pile displacement profile (not shown). The rigid pile assumption would become suitable for the weak or no bearing stratum cases. The denominators in Eq. (1) and (2) should use a value of unity.

To conclude, for the issues that what shape function shall be used in the method proposed in this study, it depends on the bearing capability of the stratum beneath a statnamic tested pile.

3.4 Applicability of the proposed method

The applicability of this wavelet based method can be evaluated by the observing the plot of wavelet coefficient versus dilation parameter of a statnamic pile. If it reveals mainly single “peak” or “undulation”, the quality of the damping ration should be good. If there appears many “peaks” in the plot, the damping ratio evaluated by the proposed method could be deteriorated.

In addition, the proposed method is applicable for all competent bearing strata, for medium strength and weak bearing stratum. For strong competent bearing strata, the triangle displacement shape function fits the displacement profile better, the denominators in Eq. (1) and (2) shall be assigned as three. For weak bearing stratum, the denominators in Eq. (1) and (2) shall be assigned as unity for the almost “rigid” pile movement and rectangle shape of displacement profile. For medium strength bearing stratum, the denominators in Eq. (1) and (2) is between three to unity, and can be calculated by integrating the trapezoid shape function. For example, the denominators are 12/7 for the trapezoid shape function in Fig. 11. The proper trapezoid shape function to be used however has to be estimated according to the strength parameters and characteristics of the bearing stratum.

4. Case Studies

To verify the proposed method in this study, two case studies of statnamic pile load tests were analyzed to show how the signals were processed and compared to the result obtained by the UPM method and the analysis results by Lin et al. (2004). There are six statnamic pile tests were performed for a building project in Taipei, Taiwan in 2002. Two of those piles were also investigated with conventional static load tests. One was an 81.1-m long pile, and the other one was 74 m long (Lin et al., 2004). The two piles were cast-in-place and inserted into the sandstone bearing layers. The soil properties of this site are shown in Table 1. Following Lin et al. (2004), the statnamic tests for the 81.1-m and 74-m piles are also denoted as Cases S81 and S74, respectively. In these two case studies, the denominators used are 12/7 as we estimate the bearing stratum of the site is of medium strength.

4.1 Case S81 - statnamic test

The diameter of the S81 pile is 1.5 m, and the length is 81.1 m. The recorded time history of
statnamic load and displacement of the pile head is shown in Fig. 12. The maximum applied statnamic load is about 19 MN, and the maximum displacement of the pile head is 17 mm. The duration of the loading is about 0.11 sec and occurs from t=0.08 sec to t=0.19 sec. The data during this duration was analyzed to determine the damping ratio and to interpret the static load-displacement curve of the pile.

The Morlet wavelet function was used to transform the time history of the load and displacement into the transformed values corresponding to the dilation parameter and time. Thus, the frequency response function can be determined by the displacement transform signals divided by the load transform signals. Because the load and displacement transform signals are complex values, the frequency response function is still a complex value. Fig. 13 illustrates the results of continuous wavelet transform to analyze the frequency response function of the statnamic test data of Case S81. In Case S81, the center frequency of 1.5 Hz and the sample period of 0.002 sec were used. The complex Morlet wavelet to obtain the complex transform values is presented as contour plots and a surface plot. The real part of transform values versus dilation parameter, $a$, and time, $t$ is shown in Fig. 13(a). Fig. 13(b) contains the imaginary part of transform values versus dilation parameter, $a$, and time, $t$. The surface plot of the modulus of the Morlet transform is shown in Fig. 13(c). In these figures, the near-zero values of the continuous wavelet transform are distributed at larger values of the dilation parameter, $a$. However, at smaller values of $a$, less than 20, and a loading time between 0.09 seconds and 0.12 seconds, we can see only one to three large undulations of peaks. Therefore, we can fairly assume that the pile-soil system is close to a SDOF in the proposed method.

Fig. 14 shows the time history of the damping ratio derived from the proposed method of this study. As expected, the damping ratio is not constant throughout the statnamic test. The damping ratio is about 0.04 near the beginning of the statnamic test and then increases to about 0.17 and 0.16 when the maximum loading and maximum displacement on the pile head were developed, respectively. The time of the maximum value of the damping ratio is 0.13 second, which is very close to when maximum loading occurred. Although non-zero values of the damping ratio of the S81 are observed when the maximum displacement is reached, the damping force could still be zero. This is because the velocity of the pile head is zero when the maximum displacement is reached, such that zero velocity multiplied by the damping ratio results in zero damping force. In Fig. 14, the damping ratio is obviously larger around 0.1 second (close to the beginning of the test), 0.13 second (close to the maximum loading) and 0.158 second (close to the maximum displacement).

Fig. 15 presents a comparison of the recorded statnamic load-displacement and the curves interpreted from the UPM, results from Lin, et al. (2004) and the proposed method of the Case S81 statnamic test. The 20 MN capacity predicted by the proposed method is lower than that of the UPM. This is because the UPM uses the damping coefficient when the maximum loading is reached, and this damping coefficient is generally larger than those of other loading durations. The UPM assumes this large damping coefficient as a constant throughout the test. This causes the UPM to overestimate the damping force, thus overestimating the static loading capacity.

The bearing capacity estimated by the proposed method is smaller than that by UPM. This may
also due to the linear shape function assumption with the reduction (7/12) of mass and damping. However, the damping obtained with the proposed method is better than that obtained with the UPM because the time varying damping can be estimated. The predicted static load curve is close to that of Lin, et al. (2004) as shown in Fig. 15. For the bearing capacity estimation, we used the simplified SDOF system with the reduced mass and damping only; while Lin, et al. (2004) used multiple DOF system with strain gage data at various depths, which requires analyses of more monitoring data and procedures.

For additional comparison, the CASE method (Bowles, 1995) was also used to evaluate the static bearing capacity, and 17 MN was calculated. This capacity is smaller than the capacity of 20 MN predicted by the proposed method.

### 4.2 Case S74 statnamic test

The measured time history of the statnamic load and displacement on the pile head of Case S74 is shown in Fig. 16. The maximum applied statnamic load is about 20 MN, and the maximum displacement on the pile head is 18 mm. The duration of the loading is about 0.1 sec and occurs from t=0.09 sec to t=0.19 sec. This duration was analyzed to determine the damping ratio and to interpret the static load-displacement curve of the S74 pile.

Similar to Case S81, Fig. 17 presents the frequency response function results of the continuous wavelet transform for Case S74 data. The real part of transform values versus dilation parameter, a, and time, t is shown in Fig. 17(a), and in Fig. 17(b) for the imaginary part. The surface plot of modulus of the Morlet transform is shown in Fig. 17(c). In these figures, for smaller values of a, less than 20, and load durations between 0.14 sec and 0.18 sec, there are some large undulations in the continuous wavelet transform results.

As shown in Fig. 18, the time history of the damping ratio derived from the proposed method is again not constant throughout the S74 statnamic test. The damping ratio is about 0.06 near the beginning of the test and then increases to about 0.3 and 0.12 when the maximum loading and maximum displacement on the pile head is developed, respectively. The time of the maximum value of damping ratio is close to the time of maximum loading near 0.118 second. The damping ratios are also larger close to the beginning of the test, at the maximum loading and at the maximum displacement, as shown in Fig. 18. This is similar to the case of S81.

The static loading capacity analyzed by the proposed method is about 24 MN which is lower than that of UPM (Fig. 19), but it is larger than that of Lin, et al. (2004), 16 MN. Because the UPM assumes a constant largest damping during statnamic loading, it could overestimate the static loading capacity in this case also. The bearing capacity of S74 is evaluated about 20 MN by the CASE method and is smaller than the 24 MN predicted in this study.

The wavelet coefficient versus dilation parameter in the three stages at the start of testing, at the maximum displacement and at the end of testing for Case S74 is shown in Fig. 20. Only one or two
larger undulations were identified on each curve. This also shows that the assumption of a SODF system for the statnamic piles in this study is reasonable.

4.3 Dynamic effect

The dynamic effect is reduced by subtracting the inertia and damping forces, as indicated in the predicted curve of this study in Fig. 15 and 19. The STN test curve displays an initially higher stiffness than the predicted results; and reversely, the STN test curve displays a lower stiffness than the predicted results when the pile starts to move upward. This phenomenon is expected due to that the inertia and damping forces of dynamic effect are subtracted to obtain the predicted results.

When the pile just begins to moving downward during the initial duration, say, moving down for about 0.04 sec, and the directions of acceleration and velocity of the pile is downward. Therefore, the inertia and damping forces are pointing upward, and shall be subtracted to eliminate the dynamic effect.

After this procedure is done, the predicted result shows a lower stiffness than that of the STN test.

Contrariously, while the pile starts to move upward after moving down, the directions of acceleration and velocity of the pile is changed to upward. Therefore, the inertia and damping forces are pointing downward, and shall also be subtracted to eliminate the dynamic effect. Therefore, the predicted result shows a higher stiffness than that of the STN test while the pile moving upward.

5. Summary and Concluding Remarks

This paper proposes a wavelet based method for the evaluation of the damping ratio in statnamic load tests by the continuous wavelet transform and half-power bandwidth method. The Morlet complex wavelet with central frequency of 1.5 Hz was used for the transform. The damping ratio variation with time is determined by finding the resonant frequency versus time, analyzed by the continuous wavelet transform. Once the damping ratio is derived, the damping force can be calculated, thus can the equivalent static load-displacement curve of the pile. A numerical simulation and two case studies of statnamic tests with static load tests were analyzed using the proposed method and compared to the results obtained from the UPM method, the CASE method, and of Lin, et al. (2004). The proposed method is validated after the comparison. Applicability of the proposed method was discussed.

Additional remarks were drawn as following:

1. The damping ratio obtained with the proposed method of this study provides a more reasonable interpretation of the static loading capacity for statnamic load test data than the UPM method does. The UPM assumes a constant damping ratio, which could overestimate the damping force, therefore predicting a higher capacity than those of the method proposed by this study.

2. The occurrence time of the maximum damping ratio is close to the occurrence time of maximum loading and/or maximum displacement.
3. The assumption of the single degree of freedom system with a linear displacement shape function in this study is proved reasonable. The reduced mass and damping could partly increase the accuracy of the bearing capacity estimation. For competent, medium strength or weak bearing stratum, the shape function shall be adjusted to obtain better description of the pile displacement profile.

4. The damping ratios obtained from the two real cases are not constant as expected. However, the damping ratios obtained from the numerical simulation of statnamic test are fairly constant. This may be due to that the Mohr-Coulomb soil model is still not able to capture the plastic and dynamic behavior of soil during the statnamic simulation.

Acknowledgement

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References


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A wavelet based method for estimating the damping ratio in Stanamic pile load tests” by Tsai, Feng and Lin

Table 1 Material properties of the pile in the numerical simulation

<table>
<thead>
<tr>
<th>Material</th>
<th>Unit mass, $\rho$ (kg/m$^3$)</th>
<th>Shear modulus, MPa</th>
<th>Bulk modulus, MPa</th>
<th>Diameter, m</th>
<th>Length, m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
<td>2400</td>
<td>9460</td>
<td>20500</td>
<td>1</td>
<td>20</td>
</tr>
</tbody>
</table>
Table 2 Material properties of the soil strata in the numerical simulation

<table>
<thead>
<tr>
<th>Soil description</th>
<th>Thickness, $m$</th>
<th>Unit mass, $\rho$, kg/m$^3$</th>
<th>Shear modulus, MPa</th>
<th>Bulk modulus, MPa</th>
<th>Cohesion, kPa</th>
<th>Friction angle, $\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>The upper soil layer</td>
<td>20</td>
<td>1715</td>
<td>2.3</td>
<td>5</td>
<td>-</td>
<td>30</td>
</tr>
<tr>
<td>The bottom soil layer</td>
<td>-</td>
<td>2156</td>
<td>7000</td>
<td>26800</td>
<td>1300</td>
<td>27.8</td>
</tr>
</tbody>
</table>
The simplified model

The displacement shape function

Fig. 1 The displacement shape function and the simplified model.
A wavelet based method for estimating the damping ratio in Stanamic pile load tests by Tsai, Feng and Lin

Layer “1”
\[ \rho = 1715\, \text{kg/m}^3 \]
\[ K = 5.0\, \text{MPa} \]
\[ G = 2.3\, \text{MPa} \]
\[ c = 0 \]
\[ \phi = 30^\circ \]

Layer “2”
\[ \rho = 2156\, \text{kg/m}^3 \]
\[ K = 26800\, \text{MPa} \]
\[ G = 7000\, \text{MPa} \]
\[ c = 1300\, \text{kPa} \]
\[ \phi = 27.8^\circ \]

Fig. 2 The axisymmetrical mesh for the numerical simulation of the statnamic test.
Fig. 3 Load and displacement versus time on the pile head of the numerical simulated pile.
A wavelet based method for estimating the damping ratio in Stanamic pile load tests” by Tsai, Feng and Lin

Fig. 4 Wavelet transform plots of the frequency response function of the numerical simulation.

(a) Contour plot of wavelet transform (real part)

(b) Contour plot of wavelet transform (imaginary part)
A wavelet based method for estimating the damping ratio in Stanamic pile load tests” by Tsai, Feng and Lin

(c) Surface plot of wavelet transform (modulus)

Fig. 4 Wavelet transform plots of the frequency response function of the numerical simulation.
Fig. 5 Damping ratio versus time of the numerical simulated pile.
Fig. 6 The result of static pile load test simulated by FLAC.
Fig. 7 Comparison of the predicted load-displacement curves for the numerical simulated pile.
Fig. 8 Vertical stress variation along the pile during the 0.12 sec static loading (increment=0.01 sec).
Fig. 9 Vertical stress versus time plot for various pile depths.
A wavelet based method for estimating the damping ratio in Stanamic pile load tests by Tsai, Feng and Lin

Fig. 10 Displacement profiles along the simulated pile normalized to the pile-head displacements at t=0.06 sec for various cohesion parameters of the bearing stratum.
A wavelet based method for estimating the damping ratio in Stanamic pile load tests by Tsai, Feng and Lin

The simplified model

The displacement shape function

Fig. 1 One kind of the trapezoid shape function assumed and the integrations for the medium strength bearing stratum.
Fig. 12 Measured load and displacement versus time on the pile head of Case S81.
A wavelet based method for estimating the damping ratio in Stanamic pile load tests by Tsai, Feng and Lin

Fig. 13 Wavelet transform plots of the frequency response function of Case S81.

(a) Contour plot of wavelet transform (real part)

(b) Contour plot of wavelet transform (imaginary part)
A wavelet based method for estimating the damping ratio in Stanamic pile load tests” by Tsai, Feng and Lin

(c) Surface plot of wavelet transform (modulus)

Fig. 13 Wavelet transform plots of the frequency response function of Case S81.
“A wavelet based method for estimating the damping ratio in Stanamic pile load tests” by Tsai, Feng and Lin

Fig. 14 Damping ratio versus time of Case S81.
Fig. 15 Comparison of the predicted load-displacement curves for Case S81.
Fig. 16 Measured load and displacement versus time on the pile head of Case S74.
(a) Contour plot of wavelet transform (real part)

Fig. 17 Wavelet transform plots of the frequency response function of Case S74.

(b) Contour plot of wavelet transform (imaginary part)

Fig. 17 Wavelet transform plots of the frequency response function of Case S74.
A wavelet based method for estimating the damping ratio in Stanamic pile load tests by Tsai, Feng and Lin

(c) Surface plot of wavelet transform (modulus)
Fig. 17 Wavelet transform plots of the frequency response function of Case S74.
Fig. 18 Damping ratio versus time of Case S74.
Fig. 19 Comparison of the predicted load-displacement curves for Case S74.
A wavelet based method for estimating the damping ratio in Stanamic pile load tests by Tsai, Feng and Lin

Fig. 20 Typical frequency response functions of Case S74.
Table 3 Summary of soil properties at Case S81 and S74 site (following Lin, et al., 2004)

<table>
<thead>
<tr>
<th>Soil description</th>
<th>Average thickness (m)</th>
<th>Unit weight (kN/m³)</th>
<th>Uncorrected SPT-N</th>
<th>Strength (total) (kPa)</th>
<th>Strength (effective) (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Backfilled layer</td>
<td>2</td>
<td>17.15</td>
<td>6</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Upper</td>
<td>6</td>
<td>17.64</td>
<td>2</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>Silty clay Middle</td>
<td>8</td>
<td>17.64</td>
<td>4</td>
<td>10</td>
<td>17</td>
</tr>
<tr>
<td>Lower</td>
<td>3.4</td>
<td>17.64</td>
<td>7</td>
<td>20</td>
<td>18</td>
</tr>
<tr>
<td>Silty clay sand/sandy silt</td>
<td>9.5</td>
<td>19.11</td>
<td>18</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Silty clay clayey silt</td>
<td>5.6</td>
<td>18.62</td>
<td>15</td>
<td>40</td>
<td>20</td>
</tr>
<tr>
<td>Silty sand/clay + gravel</td>
<td>27.4</td>
<td>19.40</td>
<td>33</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>+ weathered rock</td>
<td>27.4</td>
<td>20.58</td>
<td>&gt;50-100</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Sandstone</td>
<td>-</td>
<td>21.56</td>
<td>&gt;50-100</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>