Impacts of random scrap rate on production system in supply chain environment with a specific shipping policy

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Accepted 13 July, 2011

This paper examines impacts of random scrap rate on production system in supply chain environment with a specific n+1 distribution policy. The purpose of introducing such a specific shipping policy is to cut down inventory holding costs for both producer and customer in vendor-buyer supply chain environment. Mathematical modeling along with Hessian matrix equations is used to simultaneously determine the optimal policies for production lot-size as well as number of shipments first. Then impacts of random scrap rate on the proposed system are analyzed. Numerical example is provided to show its practical usage and to demonstrate impacts of scrap rate on the system.

Key words: Imperfect production system, random scrap rate, replenishment lot size.

INTRODUCTION

In manufacturing processes, random scrap items produced are inevitable (Nahmias, 2009). Impacts of scrap on production system must be studied specifically in order to achieve optimization of manufacturing system in terms of total production costs. Rosenblatt and Lee (1986) studied an economic production quantity (EPQ) model that deals with imperfect quality. They assumed that at some random point in time the process might shift from an in-control to an out-of-control state, and a fixed percentage of imperfect quality items are produced. Approximate solutions of optimal lot size were derived by them. Schwaller (1988) examined economic order quantity model by adding both fixed and variable inspection costs for finding and removing a known proportion of defective items in incoming lots. Wee (1993) developed and formulated an economic production policy for deteriorating items with partial back-ordering. Two numerical examples are used to illustrate the theory and computational results indicated that the proposed policy leads to lower cost. Cheung and Hausman (1997) presented an analytical model of preventive maintenance (PM) and safety stock (SS) strategies in a production environment subject to random machine breakdowns. Trade-off between investing in the two options (PM and SS) were examined, and optimality conditions under which either one or both strategies should be implemented are provided to minimize the associated cost function.

Kim et al. (2001) investigated optimal production run length and inspection schedules in a deteriorating production process. They assumed that a production process is subject to a random deterioration from the in-control state to the out-of-control state and thus produces some proportion of defective items. An optimal production run length and an optimal number of inspections that minimize total costs are derived accordingly. Grosfeld-Nir and Gerchak (2002) considered multistage production systems where defective units can be reworked repeatedly at every stage. They showed that a multistage system where only one of the stages requires

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a set-up can be reduced to a single-stage system. They proved that it is best to make the "bottle-neck" the first stage of the system and they also developed recursive algorithms for solving two- and three-stage systems. Chiu (2003) studied optimal lot size for an imperfect quality finite production rate model with rework and backlogging (Barlow and Proschan, 1965; Chiu and Chiu, 2006; Chiu et al., 2007; Kocyiğit et al., 2009; Wazed et al., 2009; Chen et al., 2010; Chiu, 2010; Chiu et al., 2010a; Wazed et al., 2010a,b; Chen and Chiu, 2011).

In real-life supply chains environment, multiple deliveries of finished products are commonly used by vendor to supply items to its buyer. Goyal (1977) examined an integrated production-inventory model for single supplier-single customer case. He proposed a method that is typically applicable to those inventory problems where a product is procured by a single customer from a single supplier. An example was provided to illustrate his proposed method. Many studies have since been carried out to address various aspects of supply chain optimization. Schwarz et al. (1985) treated the system fill-rate of a one-warehouse N-identical retailer distribution system as a function of warehouse and retailer safety stock. Approximation model was developed from a prior study to maximize system fill-rate subject to a constraint on system safety stock. Properties of fill-rate policy lines are suggested and they can be used to provide managerial insight into system optimization. Hill (1996) examined a model in which a manufacturing company purchases a raw material, manufactures a product (at a finite rate) and ships a fixed quantity of the product to a single customer at fixed and regular intervals of time, as specified by the customers, while minimizing total cost of purchasing, manufacturing and stockholding. Sarkar and Khan (1999) considered a manufacturing system that procures raw materials from suppliers in a lot and processes them into finished products which are then delivered to outside buyers at fixed points in time. A general cost model was formulated considering both raw materials and finished products. Then, using this model, a simple procedure was developed to determine an optimal ordering policy for procurement of raw materials as well as the manufacturing batch size, to minimize the total cost of meeting customer demands in time. Abdul-Jalbar et al. (2005) studied a multistage distribution/inventory system with a central ware- house and N retailers. Customer demand arrives at each retailer at a constant rate. The retailers replenish their inventories from the warehouse, which in turn orders from an outside supplier. It is assumed that shortages are not allowed and lead times are negligible. The goal was to determine policies which minimize the overall cost in the system. A heuristic procedure to compute near-optimal policies is presented with computational results on several randomly generated problems. Sarkar and Diponegoro (2009) studied optimal policy for production and procurement in a supply-chain system with multiple non-competing suppliers, a producer and multiple non-identical buyers. They assumes that manufacturer procures raw materials from suppliers, converts them to finished products and ships the products to each buyer at a fixed-interval of time over a finite planning horizon. Their objective was to determine the production start time, the initial and ending inventory, the cycle beginning and ending time, the number of orders of raw materials in each cycle, and the number of cycles for a finite planning horizon so as to minimize the system cost (Goyal and Nebebe, 2000; Sarker and Khan, 2001; Chiu et al., 2009a,b,c; Chiu and Ting, 2010; Diponegoro and Sarker, 2006; Chiu et al., 2010b,c,d,e; Chen et al., 2011). This paper reexamines and extends model of Chiu et al. (2009b) by simultaneously determining the optimal production lot size as well as optimal number of shipments for such a specific supply chain system with scrap and a cost lessening n+1 delivery policy. The joint effects of the random scrap rate and n+1 delivery policy on the production system are investigated.

**MATERIALS AND METHODS**

**Description and mathematical modeling**

Suppose in a supply chain environment, a vendor-buyer integrated production system may produce x portion of random scrap items at a production rate d. Under regular operating schedule, the constant production rate P is larger than the sum of demand rate λ and production rate of defective items d. That is: (P-d-λ)>0, where d can be expressed as d=Px. Cost parameters in this study include unit manufacturing production cost C, setup cost K per production run, vendor’s unit holding cost h, buyer’s unit holding cost h₂, disposal cost per scrap item Cós, delivery cost C₇ per item and a fixed delivery cost K₁ per shipment. Additional notation is listed as follows:

- n =number of fixed quantity installments of finished batch to be delivered to customer during tₚ, one of the decision variables to be determined for each cycle, Q= replenishment lot size, another decision variable to be determined for each cycle, T=cyat length, H=level of on-hand inventory in units, D=level of on-hand inventory in units for satisfying product demand during manufacturer’s regular production time t₁, t₁=the production uptime for the proposed model, t₂=the production uptime required for delivering the remaining perfect quality finished products, tₕ= a fixed interval of time between each installment of products delivered during t₂, l= demand during production time tₚ, Dₚ= demand during production uptime t₁, that is, D=Δt. D= Δt. I(t)= on-hand inventory of perfect quality items at time t. TC(Q,n) = total production-inventory-delivery costs per cycle for the proposed model, TC(Q,n) = total production-inventory-delivery costs per cycle for the special case model, E[TCU(Q,n)] = the long-run average costs per unit time for the proposed model, E[TCU₁(Q,n)] = the long-run average costs per unit time for the special case.

Figure 1 depicts the producer’s on-hand inventory of perfect quality items of the proposed model. Under the proposed n+1 delivery policy, an initial installment of finished products is delivered to customer for satisfying the demand during uptime t₁. At the end of production, fixed quantity n installments of the rest of finished items are delivered to customer at a fixed interval of time. Figure 2 model illustrates customer’s on-hand inventory levels of the proposed
TC(Q,n) of the proposed model consists of the setup cost, variable manufacturing cost, variable disposal cost, (n+1) fixed and variable shipping cost, holding cost for perfect quality items during \( t_1 \), holding cost for scrap items during \( t_1 \), vendor’s holding cost for finished goods during the delivery time \( t_2 \) (Figure 1), and buyer’s holding cost (Figure 2).

For derivation of the very last term (that is, the inventory quantity associated with buyer’s holding cost \( h_2 \)), one can refer to Figure 2, and first considers the area in the largest triangle, then areas in n pieces of trapezoids.

Taking into account of the randomness of scrap rate \( x \), one can use the expected values of \( x \) in cost analysis, and with further derivations one obtains \( E[TCU(Q,n)] \) as:

\[
E[TC(Q,n)] = K + CQ + C_s [xQ] + (n+1)K_r + C_f Q(1-x) \\
+ h \left[ \frac{H_t}{2} + \frac{H_1 (t_1 - t)}{2} + \frac{d_t}{2} (t_1) \right] + h \left[ \frac{n-1}{2n} H_1 t_2 \right] \\
+ h_2 \left[ \frac{H(t_1)}{2} + n \left( \frac{D+2I}{2} \right) t_n \right] \\
+ hQ \left[ \frac{1}{2} \phi \left( \frac{1}{1-x} \right) + \frac{1}{2} \phi \left( \frac{1}{1-x} \right) \right] \\
+ hQ \lambda \left[ \frac{1}{2} \phi \left( \frac{1}{1-x} \right) + \frac{1}{2} \phi \left( \frac{1}{1-x} \right) \right]
\]

Convexity of the total cost function

In order to derive the optimal production-shipment policy for the proposed model, one must first prove \( E[TCU(Q,n)] \) is convex. The Hessian matrix equations (Rardin, 1998) are employed here to verify whether the following condition (that is, Equation 3) holds or not.
\[
\left[ Q, n \right] = \left( \frac{\partial^2 E[TCU(Q,n)]}{\partial Q^2} \frac{\partial^2 E[TCU(Q,n)]}{\partial Q \partial n} \right) \left( \frac{\partial^2 E[TCU(Q,n)]}{\partial Q n^n} \right) > 0
\]

From Equation (2), one obtains the following terms:

\[
\frac{\partial E[TCU(Q,n)]}{\partial Q} = \frac{\partial^2 E[TCU(Q,n)]}{\partial Q \partial n} = \frac{\partial^2 E[TCU(Q,n)]}{\partial Q n^n}
\]

Equations (4) and (9) respectively as follows:

\[
\frac{\partial^2 E[TCU(Q,n)]}{\partial Q Q^2} = \frac{\partial^2 E[TCU(Q,n)]}{\partial Q \partial n} = \frac{\partial^2 E[TCU(Q,n)]}{\partial Q n^n}
\]

Substituting Equations (4) to (8) in (3), one has:

\[
\left[ Q, n \right] = \left( \frac{\partial^2 E[TCU(Q,n)]}{\partial Q^2} \frac{\partial^2 E[TCU(Q,n)]}{\partial Q \partial n} \right) \left( \frac{\partial^2 E[TCU(Q,n)]}{\partial Q n^n} \right) > 0
\]

Equation (9) is resulting positive, because K, K, \( \lambda \), (1-E(x)), and Q are all positive. Hence, E[TCU(Q,n)] is a strictly convex function for all Q and n different from zero. Convexity of E[TCU(Q,n)] is proved and the minimum of E[TCU(Q,n)] exists.

RESULTS

Simultaneous determination of the production-shipment policy

Once convexity of E[TCU(Q,n)] is proved, for deriving the optimal lot size Q* and the optimal number of shipments n* one can differentiate E[TCU(Q,n)] with respect to Q and with respect to n, and solve the linear system of Equations (4) and (6) by setting these partial derivatives equal to zero. With further derivations one obtains Q* and n* respectively as follows:
Further derivations one can derive jointly the optimal lot size $Q^*$ and optimal number of shipments $n^*$ as follows:

$$Q^* = \sqrt{\frac{2\lambda \left( (n+1)K + K \right)}{h \left( \frac{2\lambda^2}{P^2} - \frac{2\lambda}{P} + 1 - \frac{\lambda}{p^2} \right) + \frac{h_2 \lambda^2}{2p^2} \left( 1 - \frac{\lambda}{p} \right) + \frac{1}{p^2}}}$$  \hspace{1cm} (14)$$

And

$$n^* = \left[ \frac{K + K \left( h_2 - h \right) \left( 1 - E(x) \right)^2 + \frac{2\lambda}{P^3} \left( 1 - E(x) \right) + \frac{\lambda^2}{P^2}}{h K_1 \left[ \frac{2\lambda^3}{P^3} - \frac{2\lambda^2}{P} + \frac{\lambda}{p^2} \right] + h_2 \lambda^2 \left( \frac{2}{P^2} - \frac{\lambda}{p} \right) - \frac{h_2 Q}{2n} \left( 1 - \frac{2\lambda}{P} + \frac{\lambda^2}{P^2} \right) \right]$$  \hspace{1cm} (15)$$

To cope with the integer value of $n$, similar solution procedure (as stated earlier) should be applied to this special case for deriving the optimal $(Q^*,n^*)$.

**Numerical example with further discussion**

To enable comparisons of the models with/without random scrap, and $n$ versus $n+1$ delivery policy, the same example as in Chiu et al. (2009b) is used here. Assuming a manufacturing system has an annual production rate of 60,000 units, and a flat demand rate of 3,400 units per year. During the production, a random scrap rate is assumed to be uniformly distributed over interval $[0, 0.3]$. Cost parameters include $K_1 = $20,000 per run; $C_2 = $100; $C_3 = $20 per scrap item; $h = $20 per item per year; $K_1 = $4,400 per shipment; $C_T = $0.1 per item delivered; and $h_2 = $80 per item kept at the customer’s end per unit time.

From computation of Equation (11) one obtains $n^* = 3.84$. To determine the optimal integer value of $n$, two adjacent integer numbers and its corresponding lot-sizes (Equation 10) are plugged in Equation (2) respectively. Results are $E[TCU(Q=3073,n=3)] =$ 509,702 and $E[TCU(Q=3455,n=4)] =$ 509,012. Therefore, one obtains the optimal production lot size $Q^* = 3455$, the optimal number of deliveries $n^* = 4$, and the long-run expected cost $E[TCU(Q^*,n^*)] =$ 509,012.

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is analyzed as follows. Suppose that scrap rate $x = 0$. From computation of Equation (15) one obtains $n^* = 3.91$. To determine the optimal integer value of $n$, two adjacent integer numbers and its corresponding lot-sizes (Equation 14) are plugged in Equation (12) respectively. Results are $E[TCU(Q=2609,n=3)] =$ 437,803 and $E[TCU(Q=2938,n=4)] =$ 436,987. Therefore, one obtains the optimal production lot size $Q^* = 2938$, the optimal
number of deliveries $n^* = 4$, and the long-run expected cost $E[TCU(Q^*, n^*)] = $436.987.

This study proposes a $(n^*+1=5)$ delivery policy versus $n^* = 4$ as was used in Chiu et al. (2009b), where they had optimal $Q^* = 2652$ and the long-run expected cost $E[TCU(Q^*, n^*)] = $512,047. In theory, one would think that it costs more to have one extra shipping cost. However, total savings on inventory holding costs (both from vendor and buyer) offsets this extra expense. As a result, excluding the variable production cost (AC) and set-up cost, there is a total saving of 2.1%.

Comparing the proposed model with versus without random scrap rate, one realizes that a total savings of $72,025 when there is no scrap rate in production. It is a reduction of 14.15% in total cost.

### Conclusions

This paper investigates the impacts of random scrap rate on production system in supply chain environment with a specific $n+1$ distribution policy. Mathematical modeling along with Hessian matrix equations is used to simultaneously determine the optimal policies for production lot-size as well as number of shipments first. Impacts of random scrap rate on the proposed system are then analyzed. Numerical example is provided to show its practical usage.

The research results depict that the $n+1$ delivery policy can outperform n shipping method in terms of savings in stock holding costs from both ends of vendor and buyer. Further, impacts of random scrap rate on production system are significant in terms of extra expenses in assuring product’s quality. In summary, such a real-life system must be specifically studied in order to obtain more insights of the system parameters.

### REFERENCES
