Combining block-coded modulation codes and improved constellation extended schemes to reduce peak-to-average power ratio in orthogonal frequency-division multiplexing systems

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Abstract: Orthogonal frequency-division multiplexing (OFDM) has been recommended as the standard for numerous wireless transmission systems. However, OFDM still has deficiencies that must be improved, such as a high peak-to-average power ratio (PAPR) occurring in the transmission signal. A constellation extended scheme (CES) is one technology for reducing high PAPR. The PAPR performance of the CES is directly correlated to the amount of candidate signals, high PAPR in OFDM systems will be significantly reduced as the candidate signals increase. However, because calculating the PAPR values of candidate signals requires more inverse fast Fourier transform operations, the CES hardware circuit is difficult to achieve. Therefore this study combines the CES, partial transmit sequences (PTSs) and block-coded modulation (BCM) codes to propose a BCM–CES–PTS technology, which simultaneously improves the high computation of the CES, the necessary side information delivered in PTS and the lack of error correction capabilities in the transmitted signal itself, applying this to a 16-QAM modulation OFDM system. In addition, this study proposes two structures of the CES by arranging the extended constellation points in symmetrical and asymmetrical forms, called S-BCM–CES–PTS and AS-BCM–CES–PTS, respectively. The generator matrices of the two structures were proposed to simplify the division of the circuit into sub-blocks.

1 Introduction

Multi-carrier transmission technology [1, 2] is common not only in frequency-selective fading channels, but has also become an extremely popular wireless transmission technology. Orthogonal frequency-division multiplexing (OFDM) [3–5] is one branch of multi-carrier transmission technologies with high-spectral efficiency and simple receiver. OFDM technology has become the standard technology for numerous wireless information application systems, including the 3GPP long-term evolution technology [6], the broadband wireless access standard HIPERMAN [7] and IEEE 802.16d [8]. Each signal transmitted in an OFDM system comprises of numerous independent subcarriers. However, when the phase of each subcarrier is identical, a high peak-to-average power ratio (PAPR) occurs in the transmitted signal. A high PAPR not only decreases the efficiency of high-power amplifiers, but also limits the possible applications of an OFDM transmission system. Therefore methods to reduce a high PAPR are an extremely popular research topic related to OFDM systems [9].

Previous studies proposed combining selected mapping (SLM) with a symmetrical constellation extended scheme (CES) to effectively reduce the PAPR in OFDM systems [10]. This method can further reduce the PAPR compared with using only SLM. Briefly, the CES divides a 16-QAM signal constellation into two sets: those lacking extended constellation points (called the internal constellation set) and those possessing extended constellation points (called the external constellation set). The position of the extended constellation points in the external constellation set is assigned by the user. The extended and the internal constellation points comprise of numerous candidate sequences from which the signal with the lowest PAPR is selected as the CES transmission signal. In addition, the CES can be demodulated based on 16-QAM extended constellations, rendering the transmission of side information to return the original input data unnecessary. However, the number of candidate sequences required for the CES to effectively reduce the PAPR is relatively high. Furthermore, numerous inverse fast Fourier transform (IFFT) operations are required to calculate the PAPR values of the candidate sequences; the sequence with the lowest PAPR value becomes the transmission signal. Therefore reducing the number of operations required for the CES while preserving optimal PAPR performance remains a crucial research topic [11, 12]. Partial transmit sequences (PTSs) technology is another method of effectively reducing a high PAPR. Simply defined, PTS technology
divides the inputted data blocks into a number of non-intersecting sub-blocks of identical length, and then uses IFFT to transform these sub-blocks into time-domain signals. These transformed time-domain signals are multiplied by groups of phase factors, which are primarily used to disrupt the phases of the time-domain signals, to produce numerous candidate sequences, from which the sequence with the lowest PAPR is selected as the PTS transmitted signal. Generally, the number of IFFT operations used in PTS technology equals the number of sub-blocks. However, as the number of sub-blocks is increased to improve PAPR efficiency, the number of IFFT operations required also increases, which further complicates the system hardware. Furthermore, to ensure that the receiver can effectively restore the sent data, PTS technology requires the transmission of side information that informs the receiver which candidate sequence transmitted the signal. As the number of side information increases, the transmitted data rate decreases. Therefore how to simplify the operations of PTS technology and eliminate the need for side information has become a crucial research topic [13, 14]. However, faced with various transmission channel interferences today, CES and PTS technologies alone lack anti-noise interference capabilities.

Generally, error correcting codes, also called channel coding, are often used to solve transmission channel interference. Error correcting codes increase the redundant bits in the transmission signal; thus, when an error occurs in a transmitted signal because of transmission channel interference, the receiver only requires the redundant bits to correct the error in the failed bit. In addition to error correcting code technology, another technology that combines error correcting codes and signal modulation, namely block-coded modulation (BCM), has received significant attention. With effective bandwidth use and an easier construction method than trellis-coded modulation codes, the BCM code can be constructed simply using multilevel coding technology. This paper researched the individual CES internal and external constellation sets produced by two 16-QAM BCM codes, and simultaneously proposed the generator matrice for these two sets and a method for combining it with a PTS technology, called BCM–CES–PTS, to simplify CES operations and incorporate error correcting capabilities into CES technology. In addition, the BCM–CES–PTS technology was divided into symmetrical and asymmetrical, called S-BCM–CES–PTS and AS-BCM–CES–PTS, according to the position of the extended constellation points. Subsequently, the effects of both structures on the PAPR are simulated, analysed and compared. The BCM–CES–PTS technology divides the inputted data blocks into sub-blocks and encodes each data sub-block using BCM codes. That is, two data sub-blocks are multiplied by the generator matrix \( G_{in} \) of the internal constellation set and the generator matrix \( G_{out} \) of the external constellation set. Next, two 16-QAM BCM codewords are reorganised and combined using a random interleaver, preventing burst errors. The BCM–CES–PTS technology then divides these new sequences into numerous non-intersecting sub-blocks of identical length, where the sets without extended constellations are placed in one sub-block and sets with extended constellations are placed in another sub-block according to the differential value between the original and their extended constellation points. Using IFFT, these separated sub-blocks are transformed into time-domain signals, which are subsequently combined into groups of candidate signals by changing the phase of each sub-block. The candidate signal with the lowest PAPR is selected as the transmission signal. Compared with PTS, BCM–CES–PTS is a sub-optimal PAPR technology that uses modulated input data to adjust the calculation amount needed to lower the PAPR using BCM–CES–PTS. In addition, BCM–CES–PTS eliminates the need to transmit side information to recover the original data. In contrast to PTS, which lacks error correction capabilities, BCM–CES–PTS combines BCM codes with a modified CES based on the PTS framework and the transmitted signals for this technology possess error correction capabilities.

The rest of this paper is organised as follows: Section 2 describes the mathematical expressions of the OFDM signals and the PAPR, and introduces the PTS technology and CES technology. Section 3 explains the systematic framework of the method recommended by this paper and how it applies to 16-QAM modulation OFDM systems. The simulation results are discussed and analysed in Section 4; Section 5 provides the conclusion.

2 Research background

2.1 OFDM and PAPR

In an OFDM system, suppose \( X = (X_0, X_1, \ldots, X_{N-1}) \) is an input data block with \( N \) symbols; \( \{\phi_p(t) = e^{j\frac{2\pi}{T}pt}, p = 0, 1, 2, \ldots, N-1\} \) are defined as the continuous sine waves for modulation, where \( (1/T) \) is the bandwidth of every subcarrier. Each OFDM transmission signal \( x(t) \) with \( N \) subcarriers can be expressed as: \( x(t) = (1/N) \sum_{p=0}^{N-1} X_p \phi_p(t), 0 \leq t \leq T \). The instantaneous envelope power \( P(t) \) of signal \( x(t) \) can be written as \( P(t) = |x(t)|^2 \). The PAPR of \( x(t) \) is defined as

\[
PAPR(x(t)) = \max_{0 \leq t \leq T} \frac{P(t)}{P_{av}}
\]

where \( P_{av} \) is the average power of \( x(t) \). Generally, for an accurate approximation of the PAPR in signal \( x(t) \), discrete \( L \)-point sampling \( x(t)/L \), \( i = 0, 1, \ldots, LN - 1 \), is conducted for continuous-time signal \( x(t) \), where \( LN \)-point sampling uses \( \{X_0, X_1, \ldots, X_{N-1}\} \) and \( (N - 1) \) \( L \) zero-padding for IFFT calculation. In this paper, we applied \( L = 4 \) for numerical simulation. Assessing the effects of improving PAPR commonly employs a complementary cumulative distribution function (CCDF) for measurement. Assuming that the reference level of the PAPR is \( \text{PAPR}_0 \), then \( \text{CCDF}(\text{PAPR}) \), the probability of the PAPR is a signal being greater than \( \text{PAPR}_0 \), can be expressed as

\[
\text{CCDF}(\text{PAPR}) = P_i(\text{PAPR} > \text{PAPR}_0)
\]

2.2 Partial transmit sequences

Based on the principle of PTS, an input modulation signal block \( X \) of length \( N \) is divided into \( v \) sub-blocks of length \( N \), which satisfy \( \sum_{i=1}^{v} X_i = X \) and \( X_i \cap X_j = \phi, i \neq j \). The methods of dividing the block into sub-blocks can broadly fit three categories: adjacent partition, interleaved partition and pseudo-random partition, among which employing pseudo-random partition in PTS derives better PAPR performance than the other two methods. Next, an IFFT calculation is performed on the \( v \) sub-blocks of length \( N \),
which can be written as

\[ S_b(t) = \sum_{k=0}^{N-1} X_{ik} \phi_k(t), \quad 1 \leq i \leq v \] (2)

The time-domain signals \( S_b(t) \), \( 1 \leq i \leq v \), obtained after IFFT are called PTS. In order to create multiple candidate sequences and select the signal with the lowest PAPR for transmission, the \( v \) PTSs are multiplied by phase factors \( b = (b_1, b_2, \ldots, b_v) \) to generate \( M^v \) distinct OFDM candidate signals, which can be mathematically expressed as \( S_b(t) = \sum_{i=1}^{v} b_i S_i(t) \), where \( b_i = e^{j(2\pi \omega_i M)} \), \( \omega = 0, 1, \ldots, M - 1 \) and \( M \) is the allowed phase variation. When the number of sub-blocks \( v \) or phase variation \( M \) increases, the calculations for the PTS are more complex and search time is longer. As a result, how to obtain optimal phase factors quickly and better PAPR for transmission signals is a common topic in PTS techniques. In no-loss conditions, we can assume that \( b_1 = 1 \), reducing the number of OFDM candidate signals from \( M^v \) to \( M^{v-1} \). For the receiver to be able to restore the original input data block, the sender must transmit \( \lfloor (v-1) \log_2 M \rfloor \) bits to indicate how the signal with the smallest PAPR was selected from the OFDM candidate signals.

### 2.3 16-QAM CES technology

Fig. 1 shows a 16-QAM signal constellation and its CES using Grey mapping of the constellation. In the 16-QAM signal constellation shown in Fig. 1a, each symbol represents 4-bits of inputted data. However, in the 16-QAM CES, the 4-bits of inputted data only have two symbols that can be mapped. More precisely, when the decimal value of the 4-bits of inputted data in 16-QAM CES is \( \geq 4 \), the number of mapped symbols increases (Fig. 1b). For example, when the 4-bits of inputted data are (1, 1, 1, 1) (with a decimal value 15), they both have a mapped symbol \(-3 - j5\) and an extended symbol \(-3 + j5\). By contrast, because CES uses a modulation method to map the inputted data into two symbols, the receiver can demodulate the signal to obtain the sent data, eliminating the need to send side information to the receiver. By eliminating side information, the CES increases the data transmission rate more effectively than PTS technology. Assuming that a total of \( s \) symbols had a corresponding decimal value \( \geq 4 \), each symbol has one maximum extendable symbol, can produce \( 2^s \) candidate signals and only requires \( 2^s \) IFFT operations to determine the optimum PAPR candidate for the CES transmission signal. However, if \( s \) becomes excessive, the number of candidate signals grows exponentially, rendering the CES hardware circuit difficult to achieve. Therefore, improving the selection mechanism to ensure that the CES has fewer operations and optimal PAPR is a crucial research direction [11, 12]. Though there are two previous papers, [11 and 12], that have discussed methods for achieving acceptable bounds of PAPR performance and computational complexity within the OFDM system using the CES, neither [11] nor [12] considered the issue of Euclidean distance. The proposed CES considers both of the two issues simultaneously, which is not discussed in the previous-related works. The BCM codes are a coding mechanism designed based on the size of the Euclidean distance between symbols [15, 16]. Therefore, this paper will combine the BCM codes, CES and PTS technologies to propose a sub-optimal PAPR improvement technology with low computational complexity and a large Euclidean distance.

### 3 Recommended method

To effectively decrease the high computation of CES, this paper proposed a CES technology based on a PTS framework called CES–PTS technology. This technology not only eliminates the need to transmit side information, but also requires fewer computations, is sub-optimal and reduces the PAPR compared with CES. Although the CES–PTS technology requires less IFFT operations than CES, it lacks error correction capability under various channel noise interferences. Thus, the receiver experiences regular interference and cannot effectively determine the transmitted signal. To eliminate this deficiency, we combined BCM coding with CES–PTS technology. CES–PTS and BCM–CES–PTS are described below.

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**Fig. 1** Grey mapping of the constellation

- **a** Grey mapping of the 16-QAM signal constellation
- **b** Grey mapping of the 16-QAM constellation extension scheme
3.1 CES–PTS technology

This section introduces two types of CES–PTS frameworks. The first type is CES based on the PTS framework; its PAPR improvement performance is identical to that of CES. However, unlike CES, which requires 2^s IFFT operations to compute the PAPR of a candidate signal, CES–PTS only requires s + 1 IFFT operations, where s is the number of symbols with extended constellation points. The second type is improved CES–PTS technology, which is a sub-optimal PAPR improvement method that requires fewer computations. Though the PAPR improvement performance in the improved CES–PTS is inferior to that of CES, the improved CES–PTS requires even less than s + 1 IFFT operations. The CES–PTS technology and the improved CES–PTS technology are described below.

The CES–PTS technology first modulates the input data into symbols, and then divides the symbols into sub-blocks according to whether each symbol has extended constellation points. First, symbols without extended constellation points are partitioned into the same sub-block. Then, symbols with extended constellations are sequentially partitioned into a sub-block. This partitioning method can be considered random partitioning. In addition, the length of each sub-block is identical. Since the receiver can restore the inputted data through demodulation, the CES–PTS technology does not need to send side information. Overall, this difference in the PTS technology’s input data restoration method allows the CES–PTS technology to effectively increase the data transmission rate. For example, assume the modulated input data is X, and has s number of symbols with extended constellation points. The CES–PTS technology partitions symbols without extended constellation points into the same sub-block, X_1 (the first sub-block), and symbols with extended constellation points into s sub-blocks X_i, 2 ≤ i ≤ s + 1. To construct a CES–PTS technology based on the PTS framework, this paper defined each sub-block’s phase factor according to the presence of extended constellation points. Identical to the PTS technology, this paper first assumed that the first sub-block phase factor was b_1 = 1 and each of the other sub-block phase factors were b_i = p_i, 2 ≤ i ≤ s + 1, where p_i is the original mapped symbol and p_i is the extended constellation point in p_i. Next, I_i was assumed to represent non-zero positions in X_i, and each element in I_i was either 0 or 1. Finally, the CES–PTS technology was required to satisfy the following condition

\[ X = b_1X_1 + \sum_{i=2}^{s+1} p_iI_i = X_1 + \sum_{i=2}^{s+1} p_iI_i \]

and \( X_i \cap X_j = \phi, \quad i \neq j \) \hspace{1cm} (3)

The candidate signals produced by CES–PTS can be expressed as follows

\[ S_g(t) = b_1\text{IFFT}\{X_1\} + \sum_{i=2}^{s+1} b_i\text{IFFT}\{I_i\} = \text{IFFT}\{X_1\} \]

\[ + \sum_{i=2}^{s+1} b_i\text{IFFT}\{I_i\}, \quad b_i \in \{p_i, p_i^t\} \] \hspace{1cm} (4)

For each OFDM signal, assuming the number of mapped symbols is n and the number of symbols with extended constellation points is s and if the data is modulated according to the constellations in Fig. 1b, it becomes clear that the CES technology requires 2^s IFFT operations whereas CES–PTS only requires s + 1 operations. For example, assume that the input data content is (0, 0, 1, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1), where the mapped symbols of \((0, 0, 1, 0), (0, 0, 1, 1)\) are \((2, 1, -1, -j)\) lacking extended constellation points and the mapped symbols of \((1, 1, 1, 1)\) are \((-3 - j, -3 - j)\) possessing extended constellation points. That is, s = 2, the modulated input data can be produced as \((1 - j, -1 - j, -3 - j, -3 - j)\) after the 16-QAM modulator. If the CES technology is used, four candidate signals of X can be produced, each requiring its own IFFT operation

1. \((1 - j, 1 - j, -3 - j, -3 - j)\)
2. \((-3 - j, -3 - j)\)
3. \((-3 - j, -3 - j)\)
4. \((-3 - j, -3 - j)\)

Conversely, if the CES–PTS technology is used to partition \(X = (1 - j, -1 - j, -3 - j, -3 - j)\) into sub-blocks, X only requires three sub-blocks, whereby the content and phase of each sub-block is:

\[ X_1 = (1 - j), \quad b_1 = 1 \]
\[ X_2 = (0, 0, 1, 0), \quad b_2 = -3 - j \text{ or } -3 + j5 \]
\[ X_3 = (0, 0, 0, 1), \quad b_3 = -3 - j \text{ or } -3 + j5 \]

The same four candidate sequences produced using the CES technology can be produced from b_1X_1 + b_2X_2 + b_3X_3. In addition, the value of the phase factor only has two choices, which are the extended and non-extended symbols in each sub-block, thereby satisfying the sub-block partitioning condition of (3). The mathematical expression for this transmission signal is \(S_g(t) = \sum_{i=1}^{s+1} b_iS_{X_i}(t)\). According to the preceding example, when \(b_2 = -3 - j\) and \(b_3 = -3 - j\), the model can become the original input transmission signal. Compared with CES, the PAPR performance of the CES–PTS technology is the same as that of the CES, and the number of IFFT operations required is decreased from 4 to 3.

Though the CES–PTS technology can achieve the same PAPR performance as CES, when the number of modulated symbols, n, and symbols with extended constellations, s, increase, the number of operations required for the CES–PTS will increase in correspondence, inhibiting the realisation of necessary hardware. Therefore this paper also researches another type of sub-optimal PAPR performance to decrease the number of IFFT operations required for the CES–PTS technology, where the symbols with identical extended constellation points are separated into identical sub-blocks. Assuming that the number of symbols with identical extended constellation points is g, then the greatest number of IFFT operations the improved CES–PTS technology would require is s + 1 - g. In other words, when partitioning the symbols with identical extended constellation points into one sub-block, the CES–PTS technology can further reduce the number of IFFT operators used. For example, if the transmitted data were \((0, 0, 1, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)\) and, using Fig. 1b, s = 2 and g = 1, then the content and phase factor
for each sub-block is
\[
X_1 = (1 - j, -1 - j, 0, 0), \quad b_1 = 1 \\
X_2 = (0, 0, 1, 1), \quad b_2 = -3 - j3 \text{ or } -3 + j5
\]

Therefore the improved CES–PTS technology can reduce the number of IFFT operations required from 4 to 2. Note that the processing of candidate signal generation is identical for the CES–PTS technology and the improved CES–PTS technology. That is, the candidate signals can be generated using a linear combination of the time-domain signal with phase factors, where the time-domain signal is obtained after transforming the sub-block by IFFT operation. Finally, the transmitted OFDM signal with a low PAPR is selected from these candidate signals.

### 3.2 BCM–CES–PTS technology

This section uses Reed–Muller codes as the BCM component code in combination with the CES–PTS technology, creating a BCM–CES–PTS technology to improve the high PAPR in OFDM systems using 16-QAM modulation. Furthermore, this paper presents two structures of the BCM–CES–PTS technology based on the position of the extended constellation points in the 16-QAM constellation, symmetrical BCM–CES–PTS technology (called S-BCM–CES–PTS) and asymmetrical BCM–CES–PTS technology (called AS-BCM–CES–PTS) and simultaneously researches the generator matrices of these two technologies to apply them to the OFDM systems using the set partitioning of the 16-QAM constellation. In addition, the partitioning structure of BCM–CES–PTS differs from that of CES by using two generator matrices to produce the respective codewords for extended constellation and non-extended constellation sets. This method reduces the decision time required by the CES to determine the existence of extended constellation points, and simultaneously improves upon the lack of error correction ability in both CES and CES–PTS. To prevent burst errors from occurring at the receiving end, this paper uses a random interleaver to reorder the codewords after encoding, thus significantly reducing the data error rate at the receiving end.

Generally, each M-ary QAM BCM codeword can be constructed from a total of \( \log_2 M \) 4-QAM BCM codewords, which can be expressed as follows
\[
\theta_k = \sum_{i=1}^{\log_2 M} \left( \frac{1}{\sqrt{2}} f^{(i)} \right) e^{j(i/4)}, \quad 0 \leq k \leq N - 1 \quad \text{and} \\
c_{i,k} \in \{0, 1, 2, 3\}
\]

Using 16-QAM BCM codes as an example (see Fig. 2a), assuming that \( x = (x_0, x_1, \ldots, x_{N-1}) \) and \( y = (y_0, y_1, \ldots, y_{4N-1}) \) are two 4-QAM BCM codewords, then one 16-QAM BCM codeword can be expressed as
\[
\theta_k = \left( \frac{1}{\sqrt{2}} \right)^{x_k} + \left( \frac{1}{\sqrt{2}} \right)^{y_k} e^{j(k/4)}, \quad 0 \leq k \leq N - 1
\]

where \( \{x_k, y_k\} \in \{0, 1, 2, 3\} \).

Fig. 2b shows the CES for the AS-BCM–CES–PTS technology. If \( p_{\text{CES}} \) refers to the occurrence rate of symbols with extended constellation points generated through the data modulated, the average power of the signal constellations shown in Figs. 2a and 2b is
\[
P_{av,16-\text{QAM}} = \frac{1}{16} [0.5 \times 4 + 2.5 \times 8 + 4.5 \times 4] = 2.5
\]

and
\[
P_{av,\text{AS–BCM–CES–PTS}} = \frac{1}{16} [8 \times 2.5 + p_{\text{CES}}(4 \times 8 + 4 \times 6.5) \\
+ (1 - p_{\text{CES}})(4 \times 4.5 + 4 \times 0.5)]
= 2.5 + 2.5p_{\text{CES}}
\]

A comparison of the average power values of \( P_{av,16-\text{QAM}} \) and \( P_{av,\text{AS–BCM–CES–PTS}} \) shows that AS-BCM–CES–PTS increases the extended constellation points as well as the average power values. The external constellation sets

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**Fig. 2** CES for the AS-BCM–CES–PTS technology

- **a** Set partitioning of the 16-QAM signal constellation
- **b** Set partitioning of the 16-QAM asymmetrical CES

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(defined as constellation sets with extended constellation points) of the AS-BCM–CES–PTS technology are composed of all possible combinations of the two possible values of two 4-QAM BCM codewords, \( x_k, y_k \), where the two possible values of a 4-QAM BCM codeword are \( x_k, y_k \in \{0, 2\} \) or \( x_k, y_k \in \{1, 3\} \) simultaneously; \( (M/2) \) constellation points are internal constellation sets (defined as constellation sets without extended constellations) in the AS-BCM–CES–PTS technology. Briefly, the external constellation sets \( \Gamma = [\Gamma_1, \Gamma_2] \) for the AS-BCM–CES–PTS technology can be expressed as:

1. \( \Gamma_1 \) denotes all combinations of \( x_k, y_k \in \{0, 2\} \): \( x_k = 0, y_k = 0 \), \( x_k = 0, y_k = 2 \), \( x_k = 2, y_k = 0 \) and \( x_k = 2, y_k = 2 \).
2. \( \Gamma_2 \) denotes all combinations of \( x_k, y_k \in \{1, 3\} \): \( x_k = 1, y_k = 1 \), \( x_k = 1, y_k = 3 \), \( x_k = 3, y_k = 1 \) and \( x_k = 3, y_k = 3 \).

The above two sets can be divided into four combinations of extended constellation points. The differential value \( d_i \), \( 1 \leq i \leq 4 \), between the original constellation points and the extended constellation points for each set, can be expressed as:

1. \( \{x_k = 2, y_k = 0\}, \{x_k = 3, y_k = 1\} \): \( d_1 = -j3 \).
2. \( \{x_k = 0, y_k = 2\}, \{x_k = 1, y_k = 3\} \): \( d_2 = j3 \).
3. \( \{x_k = 1, y_k = 1\}, \{x_k = 2, y_k = 2\} \): \( d_3 = 4 \).
4. \( \{x_k = 0, y_k = 0\}, \{x_k = 3, y_k = 3\} \): \( d_4 = -4 \).

Therefore \((16/2) = 8\) symbols contain extended constellation points. Internal constellation sets are all the possible combinations of the four types of 4-QAM constellation points, \( x_k, y_k \), belonging to \( \{0, 1\}, \{1, 2\} \) and \( \{2, 3\} \), where combinations of identical codeword elements for each set of 4-QAM BCM must be eliminated. Stated more clearly, the internal constellation sets \( \Lambda = \Lambda_1, \Lambda_2, \Lambda_3, \Lambda_4 \) for the AS-BCM–CES–PTS technology can be expressed as:

1. \( \Lambda_1 \) denotes all of the combinations of \( \{x_k, y_k\} \in \{0, 1\} \): \( x_k = 0, y_k = 1 \) and \( x_k = 1, y_k = 0 \).
2. \( \Lambda_2 \) denotes all of the combinations of \( \{x_k, y_k\} \in \{0, 3\} \): \( x_k = 0, y_k = 3 \) and \( x_k = 3, y_k = 0 \).
3. \( \Lambda_3 \) denotes all of the combinations of \( \{x_k, y_k\} \in \{1, 2\} \): \( x_k = 1, y_k = 2 \) and \( x_k = 2, y_k = 1 \).
4. \( \Lambda_4 \) denotes all of the combinations of \( \{x_k, y_k\} \in \{2, 3\} \): \( x_k = 2, y_k = 2 \) and \( x_k = 3, y_k = 3 \).

where the combinations of the identical 4-QAM BCM codeword elements \( x_k = 0, y_k = 0, x_k = 1, y_k = 1, x_k = 2, y_k = 2 \) and \( x_k = 3, y_k = 3 \) must be eliminated. Fig. 3b shows the CES for the S-BCM–CES–PTS technology. The average power values of the symmetrical CES of 16-QAM, as shown in Fig. 3b, is

\[
P_{av:S-BCM–CES–PTS} = \frac{1}{16} \left[ 4(0.5 + 4.5) + p_{CES}(8 \times 6.5) \right]
\]

\[
+ (1 - p_{CES})(8 \times 2.5)
\]

\[
= 2.5 + 2p_{CES}
\]

The location of the extended constellation points influences their average power values. AS-BCM–CES–PTS is more likely to generate higher average power values than S-BCM–CES–PTS. In the S-BCM–CES–PTS technology, constellation sets \( \Gamma = \Gamma_1, \Gamma_2 \) are internal constellation sets, and constellation sets \( \Lambda = \Lambda_1, \Lambda_2, \Lambda_3, \Lambda_4 \) are external constellation sets, where the differential value between the original constellation and extended constellation for each set \( d_i, 1 \leq i \leq 4 \) can be expressed as:

1. \( \{x_k = 0, y_k = 1\}, \{x_k = 1, y_k = 0\} \): \( d_1 = -j4 \).
2. \( \{x_k = 0, y_k = 3\}, \{x_k = 3, y_k = 0\} \): \( d_2 = -j4 \).
3. \( \{x_k = 1, y_k = 2\}, \{x_k = 2, y_k = 1\} \): \( d_3 = 4 \).
4. \( \{x_k = 2, y_k = 3\}, \{x_k = 3, y_k = 2\} \): \( d_4 = j4 \).

Based on this description, this paper observed the characteristics of the constellation sets \( \Lambda = \Lambda_1, \Lambda_2, \Lambda_3, \Lambda_4 \) and \( \Gamma = \Gamma_1, \Gamma_2 \). Assuming that \( x_k = c_{i,k} + 2c_{2,k} \) and \( y_k = c_{i,k} + 2c_{4,k} \), \( 1 \leq k \leq n \), the binary description for any 16-QAM constellation can be expressed as \( (c_{i,k}, c_{2,k}, c_{4,k}, c_{1,k}) \), \( 1 \leq k \leq n \), where \( c_{i,k} \) represents the least significant bit and \( c_{4,k} \) represents the most significant bit. The characteristics for each constellation set, \( \Lambda = \Lambda_1, \Lambda_2, \Lambda_3, \Lambda_4 \) and \( \Gamma = \Gamma_1, \Gamma_2 \), are described below:

![Fig. 3 CES for the S-BCM–CES–PTS technology](image-url)

*a* Set partitioning of the 16-QAM signal constellation

*b* Set partitioning of the 16-QAM symmetrical CES
1. Constellation set $\Lambda = \{\lambda_1, \lambda_2, \lambda_3, \lambda_4\}$; in this set, the binary description for each constellation $(c_{3,k}, c_{2,k}, c_{1,k})$ possesses $c_{1,k} = c_{2,k} \oplus c_{3,k}$, where $\oplus$ represents the reverse bit of $\vee$.

2. Constellation set $\Gamma = \{\gamma_1, \gamma_2\}$; in this set, the binary description for each constellation $(c_{4,k}, c_{3,k}, c_{2,k}, c_{1,k})$ possesses $c_{1,k} = c_{3,k}$.

To enable the BCM codes to directly differentiate between internal and external constellation sets, this paper proposes a generator matrix for internal constellation sets, $G_{in}$, and a generator matrix for external constellation sets, $G_{out}$. The following describes the $G_{in}$ and $G_{out}$ for the S-BCM–CES–PTS and the AS–BCM–CES–PTS technologies. First, according to (6), one 16-QAM BCM code is composed of two 4-QAM BCM codes. Assuming that the two 4-QAM BCM codewords are $[x_k, y_k]$, $1 \leq k \leq n$, then they can be obtained by $x_k = c_{1,k} + 2c_{2,k}$ and $y_k = c_{3,k} + 2c_{4,k}$, $1 \leq k \leq n$, where $c_i = (c_{1,i}, c_{2,i}, \ldots, c_{4,i}) \in \mathbb{R}(r_i, m)$, $1 \leq i \leq 4$, is the codeword of the $r_i$th order binary Reed–Muller code. Assuming that $k_1$ and $k_2$ are the number of data bits for $RM(r_i, m/2)$ and $RM(r_{out}, m/2)$, respectively, then the $G_{in}$ and $G_{out}$ produced by the S-BCM–CES–PTS technology can be written as (see equation at the bottom of the page) and (see equation at the bottom of the page)

Among them, $G_i$ stands for the generator matrix of the Reed–Muller codes of assigned order and length, where $1 \leq i \leq 4$. $G_{in}$ is a matrix with $(k_1 + k_2 + k_3)$ columns and $2^{m/2}$ rows, and $G_{out}$ is a matrix with $(k_{a1} + k_{a2} + k_{a3})$ columns and $2^{m/2}$ rows, where $G_{in} \oplus G_{out}$ is an all-zero matrix with $a_1$ columns and $a_2$ rows. Notably, $c_1$ and $c_3$ require an identical order of the Reed–Muller code to allow $c_1$ to equal $c_3$ simultaneously. The codewords produced by $G_{in}$ and $G_{out}$ can become 16-QAM symbols through the following modulation.

\[
b_{in,k} = \left( \frac{1}{\sqrt{2}} \frac{\sqrt{j^2} + 2c_{2,k}}{2} + \sqrt{2} \frac{\sqrt{j^2} + 2c_{4,k}}{2} \right) e^{j(\pi/4)}, 0 \leq k \leq 2^{m/2} - 1
\]

and

\[
b_{out,k} = \left( \frac{1}{\sqrt{2}} \frac{\sqrt{j^2} + 2c_{2,k}}{2} + \sqrt{2} \frac{\sqrt{j^2} + 2c_{4,k}}{2} \right) e^{j(\pi/4)}, 0 \leq k \leq 2^{m/2} - 1
\]

The codewords with a length of $2^m$ produced by the BCM–CES–PTS technology can be expressed as

\[
\begin{align*}
G_{in} &= \begin{bmatrix}
G_1 \in \mathbb{R}(r_1, m/2) & 0_{k_1 \times 2^{m/2}} & 0_{k_2 \times 2^{m/2}} & 0_{k_3 \times 2^{m/2}} & G_3 \in \mathbb{R}(r_3 = r_1, m/2) & 0_{k_1 \times 2^{m/2}} & 0_{k_2 \times 2^{m/2}} & G_4 \in \mathbb{R}(r_4, m/2) & 0_{k_1 \times 2^{m/2}} & 0_{k_2 \times 2^{m/2}} & 0_{k_3 \times 2^{m/2}} & \end{bmatrix} \\
&= (k_1 + k_2 + k_4) \times 2^{m/2}
\end{align*}
\]

\[
G_{out} = \begin{bmatrix}
G_1 \in \mathbb{R}(r_{a1}, m/2) & 0_{k_1 \times 2^{m/2}} & 0_{k_2 \times 2^{m/2}} & 0_{k_3 \times 2^{m/2}} & G_3 \in \mathbb{R}(r_{a3} = r_{a1}, m/2) & 0_{k_1 \times 2^{m/2}} & 0_{k_2 \times 2^{m/2}} & G_4 \in \mathbb{R}(r_{a4}, m/2) & 0_{k_1 \times 2^{m/2}} & 0_{k_2 \times 2^{m/2}} & 0_{k_3 \times 2^{m/2}} & \end{bmatrix} \\
= (k_{a1} + k_{a2} + k_{a3}) \times 2^{m/2}
\]

To avoid burst errors occurring during channel interference, the method proposed by this paper used a random interleaver to reorder the BCM–CES–PTS codewords, thus improving burst error issues. The BCM–CES–PTS code rate can be expressed as

\[
R = \frac{(k_1 + k_2 + k_4) + (k_{a1} + k_{a2} + k_{a3})}{8 \times 2^m}
\]

Since $k_3 = k_4$ and $k_{a3} = k_{a4}$, the code rate will decrease but the error correction capability will increase dramatically because of repeated transmission of the same data bits.
The BCM–CES–PTS technology first encodes the input data into internal or external constellation sets. Then, the 16-QAM CES modulation determines which modulated symbols to partition into a sub-block, where the symbols lacking extended constellations are placed in the same sub-block and symbols with identical differential values between their original constellation and extended constellation are grouped in the same sub-block. During sub-block partitioning, the BCM–CES–PTS technology will also remember the non-zero positions for each sub-block containing extended constellations and produce another phase factor with an identical non-zero position, the contents of which are the differential value between the original constellation points and the extended constellation points in the corresponding sub-block. Assuming that the number of symbols with identical extended constellation points is \(g\). More clearly, the BCM–CES–PTS technology partitions the encoded signals into \(s-g\) sub-blocks of external constellation sets according to the improved CES–PTS technology, and transforms all the \(s+1-g\) sub-blocks into time-domain signals using IFFT operations. The mathematical expression for this technology is described below.

\[
S_b(t) = \sum_{i=1}^{s+1-g} b_i S_{b_i}(t) = \text{IFFT}\{X_1\} + \sum_{i=2}^{s-g+1} \text{IFFT}\{X_i\} \oplus \text{IFFT}\{D_i\} = \text{IFFT}\{X_1\} + \sum_{i=2}^{s+1-g} K_i
\]

where the non-zero position for elements \(D_i\) and \(X_i\) is identical, but the internal elements of \(D_i\) are the differential values, \(d_i\), between the original constellation and extended constellation corresponding to \(X_i\). To be more specific, all non-zero elements in \(X_i\) are constellation points with extended constellation points, and \(D_i\) represents the difference vector resulting from converting \(X_i\) into extended constellation points; where the position of the non-zero element \(d_i\) is identical to that of the element that has its extended constellation point in vector, and the value of \(d_i\) is the difference in distance between the constellation point and the extended constellation point of the corresponding position in \(X_i\). In addition, \(b_i \in [0, 1]\), \(2 \leq i \leq s-g+1\) and \(K_i\) satisfies

\[
K_i = \begin{cases} 
\text{IFFT}\{X_i\} + \text{IFFT}\{D_i\}, & \text{if } b_i = 1 \\
\text{IFFT}\{X_i\}, & \text{if } b_i = 0 
\end{cases}
\]

As the need for phase change in the BCM–CES–PTS technology is determined by the value of \(b_i\), the BCM–CES–PTS replaces the multiplier used in traditional PTS technology to alter the transmission signal phases with an adder. Next, the BCM–CES–PTS produces \(2^{s+g}\) candidate sequences, and selects the sequence with the lowest PAPR value to be the transmission signal. In the BCM–CES–PTS, the phase factors used in PTS technology changed the multiplier with the adder, increasing the \(s\) operations of IFFT, which only require execution once. In summary, for each OFDM signal transmitted, the CES technology requires \(2^s\) IFFT operations, whereas the CES–PTS technology requires \((s+1-g)\) IFFT operations and the BCM–CES–PTS technology requires \([s+1-g]+s\) IFFT operations, where the extra \(s\) operations required compared with CES–PTS are primarily used to produce the necessary phase factors for candidate sequences. Finally, the BCM–CES–PTS technology linearly combines the time-domain signals and the phase factor to produce more candidate signals from which the signal with the lowest PAPR will be selected as the transmission signal.

![Fig. 4](image-url)

**Fig. 4** Comparison of the PAPR statistics of different generator matrices, \(G_{in}\) and \(G_{out}\), for a 16-QAM modulation OFDM, using symmetrical BCM–CES–PTS, technology system.
4 Simulation results

To effectively evaluate the PAPR reduction by the BCM–CES–PTS technology, this paper applied the technology to an OFDM system that uses 16-QAM modulation and possesses \( N = 256 \) sub-carriers. In the simulation results, 100,000 OFDM data blocks were randomly produced. To calculate the PAPR value of each signal accurately, four times oversampling was conducted to approximate the true PAPR. As both BCM–CES–PTS and CES–PTS use random sub-block partitioning, the simulation results were compared with the PTS technology using random partitioning.

To compare the respective PAPR improvement performance for symmetrical and asymmetrical BCM–CES–PTS technology in an OFDM system, this paper differentiated between different levels of the Reed–Muller code to construct the S-BCM–CES–PTS technology and AS-BCM–CES–PTS technology, and also compared various combinations of the generator matrices \( G_{\text{in}} \) and \( G_{\text{out}} \). For each OFDM symbol period, PTS generates \( 2^5 = 32 \) candidate sequences and the S-BCM–CES–PTS and AS-BCM–CES–PTS generate up to 32 candidate sequences. In Fig. 4, the S-BCM–CES–PTS technology not only reduces the PAPR of the traditional OFDM system by over 2 dB, it also provides error correction ability and effective bandwidth use. Although the S-BCM–CES–PTS is a sub-optimal PAPR reduction technology compared with PTS without error correction ability, side information is not required and its transmission signal can enhance error correction by selecting the Reed–Muller code. Furthermore, if \( G_{\text{in}} \) is fixed, the error correction ability decreases slightly, whereas when the level of \( G_{\text{out}} \) increases continuously, the PAPR decreases. The two curves for the AS-BCM–CES–PTS technique in Fig. 5 used \( r_1 = r_3 = 0, r_2 = 2, r_4 = 3 \) and \( m = 8 \) for constructing the generator matrix \( G_{\text{in}} \); the parameters for the generator matrix \( G_{\text{out}} \) are shown in Fig. 5. In Fig. 5, when both asymmetrical and symmetrical BCM–CES–PTS technologies lack error correction capability, the asymmetrical structure is more capable of effectively improving the OFDM PAPR than the symmetrical structure. In addition, under a fixed \( G_{\text{in}} \), the AS-BCM–CES–PTS technology will have similar characteristics to the S-BCM–CES–PTS technology, meaning its PAPR improvement capabilities will increase as the level of \( G_{\text{out}} \) increases.

5 Conclusion

This paper combined BCM codes, CES and PTS technologies to propose a sub-optimal PAPR improvement technology called BCM–CES–PTS. The BCM–CES–PTS technology constructs CES extended constellation sets using BCM codes, simultaneously incorporates error correction abilities and low computations in CES–PTS and can be applied to 16-QAM modulation OFDM systems. Compared with the PTS technology, the BCM–CES–PTS technology is a sub-optimal PAPR improvement technology. In addition, its transmission signal is not required to send side information, and it possesses error correction abilities. Compared with the CES technology, BCM–CES–PTS improves upon the high computations required by the CES. This paper also presented the symmetrical and asymmetrical BCM–CES–PTS technologies and the generator matrices for the extended constellation sets produced by these two technologies, thereby simplifying the sub-block partitioning circuit. In the simulation results, the asymmetrical BCM–CES–PTS was not only superior to the symmetrical technology in reducing PAPR, but it also did not need to send side information, had low computation and possessed error correction abilities.

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