ANALYSIS OF A STANDBY REDUNDANT SYSTEM WITH CONTROLLED ARRIVAL OF FAILED MACHINES

Abstract- System’s reliability and availability are an important issue in power plants, manufacturing systems, and industrial systems. This study investigates the issues of reliability and availability in the machine repair system. A controlling arrival policy is employed to maintain the repair quality of maintenance facility. The operation of control policy is when the number of failed machines reaches the maintenance facility’s capacity, no further arriving failed machines are allowed to enter the maintenance facility until the maintenance facility capacity decreases to a certain threshold value $F$. For such system, we derive the explicit expressions for the steady-state availability, the steady-state busy period, and the mean time to system failure. We also perform a parametric analysis for investigating the effect of changes in the system parameters on the total expected profit function and the cost effectiveness function. Several cases are analyzed graphically to study the effect of various parameters on the reliability and mean time to failure (MTTF) of the system.

Keywords- Controlling arrival policy, steady-state availability, busy period, mean time to failure
1. INTRODUCTION

System’s reliability and availability are an important issue in power plants, manufacturing systems, and industrial systems. In many industrial processes, production machines are unreliable and may break down while they are operating. When a machine breaks down, it is sent to the maintenance facility to be repaired by technicians. To enhance the system reliability and availability, most of the plants keep standby machines, which can substitute the failed machines, for maintaining the production quality and reducing the loss of production capacity. In this paper, we study the reliability and availability characteristics of a repairable system (consisting of $M$ operating machines and $S$ warm standbys) with a control arrival policy ($F$-policy).

System failure is defined as when the number of operating machines be less than $K$ operating machines where $K=1,2,3,\ldots,M$ ( $K$ out of $M+S$ systems). Therefore, if $n$ denotes the number of failed machines in the system, the system is failed if and only if $n \geq L = M + S - K + 1$, where $L$ denotes the number of machines in the system. In addition, $F$-policy means that when the number of failed machines reaches the maintenance facility’s capacity $N$, no further arriving failed machines are allowed to enter the system until the system capacity decreases to a certain threshold value $F$.

Controllable queueing model is used to find the optimal operating policy. Past works on such queueing model could be divided into two parts, controlling service and controlling arrival. The researches on controlling service include $N$-policy introduced by Yadin and Naor [1], $T$ policy proposed by Heyman [2], and $D$-policy introduced by Balachandran [3]. Among those operating policies, researches on $N$-policy queueing models gain a lot of attention because the analytical process is easily tractable. The extensions and variations of these vacation models with $N$-policy can refer to Lee et al. [4, 5], Ke[6], Amuganathan and Jeyakumar [7], Choudhury and Madan [8] and others. The developments and applications on service control of queueing systems are rich and varied (see Tadj and Choudhury [9]). On the other side, the pioneering work in the issue of controlling arrival, $F$-policy, was first investigated by Gupta [10]. Ke et al. [11] studies the operating characteristics of an $M/G/1$ queuing system with a randomized control policy ($<p, N>$-policy) and at most $J$ vacations. Yang et al. [12] studied the optimal control of a randomized control policy ($<T, p>$-policy) $M/G/1$ queue with second optional service and general startup times in which the server is typically subject to unpredictable breakdowns.

Regarding machine repair system, Ke and Wang [13] analyzed it with constant balking probability, negative exponential distributed reneging, and unreliable servers. A subsequent study, by Wang and Ke [14], revised this model with reneging behavior. System steady-state availabilities and some system performance measures were presented in their research. Wang, et al. [15] developed the profit analysis of the $M/M/R$ machine repair problem with balking, reneging, and standby switching failures. By employing the direct search method and the steepest descent method, the global maximum values can be determined under system constraints. Some profit analyses of cold standby system can be found in [16][17][18]. Besides, Ke and Lin [19] modeled manufacturing systems using two queueing systems with different repair rates and different numbers of technicians. A comprehensive and exhaustive discussion of machine repair problems was given by Haque and Armstrong [20].
In the paper, we investigate a $F$-policy $M/M/1/N$ machine repair system, where $N$ is equal to $M+S$, and derive the explicit expressions for the steady-state availability, $A(\infty)$, the steady-state busy period, $B(\infty)$, and the mean time to system failure (MTTF). To the best of our knowledge, there has been no research on such system. Besides the lack of research works on such model, our study is also motivated by some practical applications. For example, printed circuit boards are known as PCBs, which are electronic circuits created by mounting electronic components on a nonconductive board. PCBs have been in widespread use, mainly in electrical equipment, such as cell phones, video games, DVD players, calculators, computers, etc. In order to increase production efficiency, mass produced PCBs manufactured are allowed. Generally, mass production of the PCBs is almost manufactured by robots. In such application, we consider a PCB manufacturer with some unreliable robots and standby robots which can substitute the failed robots. The robot may break down at any time while it is working. When the robot is broken down, it is immediately sent to maintenance facility with finite capacity and repaired, where the repair time follows an exponential distribution. At the same time, a standby robot will be activated to substitute the failed robot. To maintain the repair quality of maintenance facility, when the capacity of maintenance facility reached to the predefined loading level, the failed robots do not be allowed to enter the system until the number of failed robots in maintenance facility decreases to a threshold level. An interesting issue raised by this case is to perform parametric research which provides numerical results to show the effects of various system parameters on the expected profit function and the cost effectiveness function.

The rest of this paper is organized as follows. In Section 2, we present the investigated queueing model. The mathematical model and its analytical steady-state solutions are developed in Section 3. We also derived the explicit expressions for the steady-state availability, $A(\infty)$, the steady-state busy period, $B(\infty)$ and the mean time to system failure, MTTF. In Section 4, we develop a cost effectiveness function and the total expected profit function per unit time for the presented model, and presented various system performance measures. The numerical illustration is provided in Section 5 and then some concluding remarks are drawn.

2. MODEL DESCRIPTIONS

We study the reliability and availability issues of the $F$-policy $M/M/1/N$ machine repair system consisting of $M$ operating machines and $S$ warm standbys. The assumptions and notations are defined as follows:

1. Each operating machine fails independent of the state of the others and has an exponential time-to-failure distribution with parameter $\lambda$. Whenever an operating machine breaks downs, it is instantly replaced by a standby machine.

2. Each available standby machine fails independent of the state of all the others and has an exponential time-to-failure distribution with parameter $\alpha$ ($0 \leq \alpha \leq \lambda$).

3. Whenever a machine fails, it is immediately sent to the repair facility where the repair work is provided in the order of breakdowns. The time–to-repair of failed machine follows the exponential
distribution with parameter $\mu$.

4. We assume that the switch between failed machine and standby machine is perfect and switchover time is instantaneous.

5. When the system permits accumulation of failed machines, the server requires an exponential startup time with parameter $\gamma$.

6. Arriving failed machines form a single waiting line based on the order of their arrivals.

7. We assume that when a standby machine moves into an operating state, its characteristics will be that of an operating machine.

8. If a machine repair system reaches its finite capacity $L$ ($L < \infty$), no further failed machines are allowed to enter the maintenance facility until enough failed machines who are already in the maintenance facility have been repaired so that the number of failed machines decreases to a predetermined threshold $F$ ($0 \leq F \leq L - 1$).

3. STEADY STATE RESULTS

In this section, we study the investigated system’s availability, busy period, and system’s reliability.

3.1. Availability analysis of the system

The reliability function of the investigated system, at $t = 0$, can be developed through the birth and death process. Let $p_{n,0}(t)$ be the probability of $n$ failed machines in the system at time $t$ ($t \geq 0$) when the arrivals are not allowed to enter into the system, where $n = 0, 1, 2, \ldots, L$; $p_{n,1}(t)$ is the probability of $n$ failed machines in the system at time $t$ ($t \geq 0$) when the arrivals are allowed to enter into the system, where $n = 0, 1, 2, \ldots, L$. We also let $P(t)$ denote the probability vector at time $t$, then the initial conditions for this problem are

\[
P(0) = [p_{1,0}(0), \ldots, p_{1,L-1}(0), p_{0,0}(0), p_{0,1}(0), \ldots, p_{0,L}(0)]^T = [1, 0, \ldots, 0]^T
\]

where $L = M + S - K + 1$ denotes the total number of machines in the system.

The failure rate $\lambda_n$ for this system is given by

\[
\lambda_n = \begin{cases} 
M\lambda + (S - n)\alpha & n = 0, 1, \ldots, S \\
(M + S - n)\lambda & n = S + 1, \ldots, L - 1 \\
0 & otherwise
\end{cases}
\]
By omitting the argument \( t \) in \( p_{i,a}(t) \), so that \( p_{i,a}(t) \equiv p_{i,a} \), and employing the method of linear first order differential equations for Figure 1, we obtain the following differential equations:

\[
\frac{dp_{1,0}}{dt} = -\lambda_0 p_{1,0} + \mu p_{1,1} + \gamma p_{0,0},
\]

(1)

\[
\frac{dp_{1,n}}{dt} = \lambda_{n-1} p_{1,n-1} - (\lambda_n + \mu) p_{1,n} + \mu p_{1,n+1} + \gamma p_{0,n},
\]

\( 1 \leq n \leq F \)

(2)

\[
\frac{dp_{1,n}}{dt} = \lambda_{n-1} p_{1,n-1} - (\lambda_n + \mu) p_{1,n} + \mu p_{1,n+1},
\]

\( F + 1 \leq n \leq L-2 \)

(3)

\[
\frac{dp_{1,L-1}}{dt} = \lambda_{L-2} p_{1,L-2} - (\lambda_{L-1} + \mu) p_{1,L-1},
\]

(4)

\[
\frac{dp_{0,0}}{dt} = -\gamma p_{0,0} + \mu p_{0,1},
\]

(5)

\[
\frac{dp_{0,n}}{dt} = - (\mu + \gamma) p_{0,n} + \mu p_{0,n+1},
\]

\( 1 \leq n \leq F \)

(6)

\[
\frac{dp_{0,n}}{dt} = - \mu p_{0,n} + \mu p_{0,n+1},
\]

\( F + 1 \leq n \leq L-1 \)

(7)

\[
\frac{dp_{0,L}}{dt} = - \mu p_{0,L} + \lambda_{L-1} p_{1,L-1},
\]

(8)

Let

\[
P = \begin{bmatrix}
\frac{dp_{1,0}}{dt}, & \frac{dp_{1,L-1}}{dt}, & \frac{dp_{0,0}}{dt}, & \frac{dp_{0,1}}{dt}, & \ldots, & \frac{dp_{0,L}}{dt}
\end{bmatrix}^T
\]

then Equation(1)-(8) can be written in matrix form as

\[
\dot{P} = Q_a P
\]

where

\[ Q_a = \]
The steady-state availability can be obtained using the following procedure. In the steady-state, the derivatives of the state probabilities become zero. Then the steady-state availability can be calculated as:

\[ A(\infty) = 1 - P_{0,1}(\infty) \]  

(9)

and

\[ Q_0 P(\infty) = 0 \]  

(10)

where \( 0 \) is the column zero vector.

For this case, the normalizing condition is

\[ \sum_{n=0}^{L-1} p_{1,n}(\infty) + \sum_{n=0}^{L} p_{0,n}(\infty) = 1 \]  

(11)

We substitute Equation (11) into any one of the redundant rows in Equation (10) to yield

\[ Q = \begin{pmatrix} p_{1,0}(\infty) \\ p_{1,1}(\infty) \\ \vdots \\ p_{1,L-1}(\infty) \\ p_{0,0}(\infty) \\ \vdots \\ p_{0,1}(\infty) \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix} \]  

(12)

where \( Q = \)
The solution of (12) provides the steady-state probabilities in the availability case. The explicit expression for \( A(\infty) \) is given by

\[
A(\infty) = 1 - p_{0,0}(\infty) = 1 - \frac{1}{(L - F) + \frac{\mu F_{i+1}}{\gamma (\mu + \gamma)} + \sum_{i=1}^{F} \left( \frac{\mu}{\mu + \gamma} \right)^{F_{i+1}} + \mu \sum_{i=1}^{L} \frac{\delta_{i-1,j}}{\Delta_{i-1,j}} - \frac{\mu}{\mu + \gamma} \sum_{i=1}^{K} \frac{\delta_{i-1,j}}{\Delta_{i-1,j}} \left( \frac{\mu}{\mu + \gamma} \right)^{F_{i+1}}}
\]

(13)

where

\[
\delta_{K,j} = \begin{cases} 
\mu \delta_{K,j+1} + \Delta_{K,j} & 0 < i < K \leq L - 1 \\
\mu + \Delta_{K,K} & i = K \\
1 & i > K 
\end{cases}
\]

(14)

\[\Delta_{K,j} = \prod_{i=j}^{K} \lambda_{i}, \quad 0 \leq i \leq K \leq L - 1\]

3.2. Busy period analysis of the system

The steady-state busy period can be calculated as

\[
B(\infty) = 1 - p_{0,0}(\infty) - p_{1,0}(\infty) - p_{0,1}(\infty)
\]

(15)

and the explicit expression for \( B(\infty) \) is given by

\[
B(\infty) = 1 - p_{0,0}(\infty) - p_{1,0}(\infty) - p_{0,1}(\infty)
= 1 - \left\{ 1 + \gamma \left( 1 + \frac{\delta_{L-1,1}}{\Delta_{L-1,0}} \right) \left( \frac{\mu + \gamma}{\mu} \right)^{F_{L+1}} - \frac{\gamma}{\mu} \sum_{k=1}^{F_{L+1}} \frac{\delta_{k-1,1}}{\Delta_{k-1,0}} \left( \frac{\mu + \gamma}{\mu} \right)^{k-1} \right\} p_{0,0}(\infty)
\]

(16)

where
\[
P_{o,\delta}(\infty) = \frac{1}{1 + \gamma \left[ (L - F) + \frac{(\mu + \gamma)}{\mu} \sum_{i=1}^{\infty} \frac{\delta_{L-i_j}}{\Delta_{L-i_j}} + \frac{\gamma}{\mu} \sum_{k=1}^{\infty} \sum_{i=1}^{\infty} \frac{\delta_{K-i_j}}{\Delta_{K-i_j}} \right]}
\]

\[\delta_{K,j}, \Delta_{K,j} \text{ are defined above}\]

### 3.3. Reliability analysis of the system

Because the initial conditions of reliability analysis are the same as the availability case,

\[
P(0) = [p_{1,0}(0), ..., p_{1,L-1}(0), p_{0,0}(0), p_{0,1}(0), ..., p_{0,L}(0)]^T = [1, 0, ..., 0]^T,
\]

hence the differential equations form for this problem can be expressed as

\[
\dot{P} = Q_r P
\]

where

\[Q_r = \]

However, the evaluation of the transient solution is very complex. To calculate the MTTF, we take the transpose matrix of \(Q_r\) and delete the rows and columns for the absorbing state(s). The new matrix is denoted as B. The expected time to reach an absorbing state is calculated from
where $B =$

\[
\begin{bmatrix}
\lambda_0 & \lambda_0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 \\
\mu - (\lambda_0 + \mu) & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 \\
0 & \mu & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & 0 & \cdots & \lambda_{r-1} & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 \\
0 & 0 & \cdots & - (\lambda_r + \mu) & \lambda_r & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 \\
0 & 0 & \cdots & \mu & - (\lambda_{r+1} + \mu) & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & 0 & \cdots & 0 & 0 & \cdots & \lambda_{L-2} & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & \cdots & - (\lambda_{L-1} + \mu) & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 \\
\gamma & 0 & \cdots & 0 & 0 & \cdots & 0 & - \gamma & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 \\
0 & \gamma & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & \mu - (\mu + \gamma) & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
0 & \gamma & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & \gamma - (\mu + \gamma) & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & \cdots & \gamma & 0 & \cdots & 0 & 0 & \cdots & 0 & \mu - \mu & \cdots & 0 & \mu - \mu & \cdots & 0 & \mu - \mu & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & \mu - \mu \\
\end{bmatrix}
\]

This method is successful because of the following relations:

\[
E[T_{P(0) \rightarrow P\text{(absorbing)}}] = P(0) \int_0^\infty e^{Bt} dt
\]

and \( \int_0^\infty e^{Bt} dt = -B^{-1} \)

Thus the following explicit expression for the MTTF is obtained:

\[
E[T_{P(0) \rightarrow P\text{(absorbing)}}] = MTTF = \sum_{i=1}^{L} \frac{\delta_{L-1,i}}{\Delta_{L-1,i-1}}
\]

where \( \delta_{L-1,i} = \begin{cases} 
\frac{\mu \delta_{L-1,i+1} + \Delta_{L-1,i}}{\mu + \Delta_{L-1,L-1}} & 0 < i < L - 1 \\
1 & i = L - 1 \\
\Delta_{L-1,i} = \prod_{j=1}^{L-1} \lambda_j, & 0 \leq i \leq L - 1
\end{cases} \)
4. SYSTEM PERFORMANCE MEASURES

Using the steady-state probabilities derived in the previous section, we compute important system performance measures of the F-policy M/M/1/N machine repair system. The assumptions and notations are defined as follows:

\[ P_F \equiv \text{the probability that the server requires a startup time before starting the service;} \]

The expression for \( P_F \) is given by:

\[ P_F = \sum_{n=0}^{\infty} P_{0_n} (\infty), \quad (19) \]

The effective arrival rate that the server starts the service after startup is given as follows

\[ \lambda_F = \sum_{n=0}^{F} \lambda_n P_{1_n} (\infty). \]

Using the definitions of these elements listed above, the total expected profit function per unit time is determined by:

\[
\text{profit}(\infty) = C_0 A(\infty) - C_1 B(\infty) - C_2 \lambda_F P_F
\]

where \( C_0 \): the revenue per unit time when one failed machine joins the system;

\( C_1 \): the cost per unit time when the system is under repair;

\( C_2 \): the setup cost per unit time for preparatory work of the server before starting the service;

We also define the cost effectiveness as \( \frac{\text{Availability}}{\text{Total cost}} \) to be an alternative cost criterion. This criterion is useful for the effective use of available resource.

\[
Ce = \frac{A(\infty)}{\text{Total cost}} = \frac{A(\infty)}{C_1 B(\infty) + C_2 \lambda_F P_F}
\]

(21)
5. NUMERICAL ILLUSTRATION

In this section, we provide some numerical examples to study the effects of system parameters on the total expected profit and the cost effectiveness. For convenience, all numerical experiments are performed by considering the following cost parameters elements as: $C_0 = 400/\text{day}, C_1 = 100/\text{day}, C_2 = 30/\text{day}$. First, we set the system’s parameters as $M = 5, S = 3, \lambda = 0.8, \alpha = 0.05, \gamma = 0.1$. The scatterplot of profit versus $F$ and $K$ is shown in Figure 2. It appears from Figure 2 that the expected profit function decreases as the threshold level $F$ increase. The result of the cost effectiveness $(C_e)$ is depicted in Figure 3. We also obtained the same conclusion.

![Scatterplot of profit vs K and F](image1)

Figure 2. Scatterplot of profit versus $F$ and $K$

![Scatterplot of Cost effectiveness(Ce) vs K and F](image2)

Figure 3. Scatterplot of $C_e$ versus $F$ and $K$
Next, we perform the parametric analysis to investigate the effect of changes in the system parameters on the total expected profit function. We set the system’s parameters as \( M = 5 \) and the values of \( \mu \) from 0.5 to 5.0. The results of various system parameters on the total expected profit function are depicted in Figures 4–9 for Cases 1–6, respectively.

Cases 1: \( M = 5, S = 3, F = 2, K = 2, \lambda = 0.8, \gamma = 0.1 \), and vary the values of \( \alpha \) from 0.05 to 0.35.

Cases 2: \( M = 5, S = 3, F = 2, K = 2, \lambda = 0.8, \alpha = 0.05 \), and vary the values of \( \gamma \) from 0.05 to 0.2.

Cases 3: \( M = 5, S = 3, F = 2, K = 2, \alpha = 0.05, \gamma = 0.1 \), and vary the values of \( \lambda \) from 0.6 to 1.2.

Cases 4: \( M = 5, F = 2, K = 2, \lambda = 0.8, \alpha = 0.05, \gamma = 0.1 \), and vary the values of \( S \) from 1 to 4.

Cases 5: \( M = 5, S = 3, K = 2, \lambda = 0.8, \alpha = 0.05, \gamma = 0.1 \), and vary the values of \( F \) from 0 to 3.

Cases 6: \( M = 5, S = 3, F = 2, \lambda = 0.8, \alpha = 0.05, \gamma = 0.1 \), and vary the values of \( K \) from 1 to 4.

From Figures 4–9, we can observe that (1) The total expected profit almost don’t change when the failure rate of warm standbys \( \alpha \) increase; (2) The values of \( \gamma \) (startup rate) increase, the profit decrease; (3) The failures rate of operating machine \( \lambda \) increase, the profit increase; The values \( \lambda \) affects profit significantly when the repair rate varies; (4) The number of warm standby \( (S) \) has a slight effect on the total expected profit; and (5) The total expected profit decrease when the threshold level \( F \) increases. From these figures, the reveal that the profit increase first and then decrease when the repair rate \( \mu \) increase from 0.5 to 5.0.

![Figure 4. Profit versus repair rate \( \mu \), with \( M = 5, S = 3, F = 2, K = 2, \lambda = 0.5, \gamma = 0.1 \)](image1)

![Figure 5. Profit versus repair rate \( \mu \), with \( M = 5, S = 3, F = 2, K = 2, \lambda = 0.5, \alpha = 0.05 \)](image2)
Finally, we analyze the effects of various system parameters on the cost effectiveness. The results are depicted in Figures 10~15 for Cases 7~12, respectively.

Cases 7: $M = 5, S = 3, F = 2, K = 2, \lambda = 0.8, \mu = 2.0$, and vary the values of $\alpha$ from 0.05 to 0.35.

Cases 8: $M = 5, S = 3, F = 2, K = 2, \lambda = 0.8, \alpha = 0.05$, and vary the values of $\mu$ from 1.0 to 4.0.

Cases 9: $M = 5, S = 3, F = 2, K = 2, \mu = 2.0, \alpha = 0.05$, and vary the values of $\lambda$ from 0.6 to 1.2.

Cases 10: $M = 5, F = 2, K = 2, \lambda = 0.8, \alpha = 0.05, \gamma = 0.1$, and vary the values of $S$ from 1 to 4.

Cases 11: $M = 5, S = 3, K = 2, \lambda = 0.8, \alpha = 0.05, \gamma = 0.1$, and vary the values of $F$ from 0 to 3.
Cases 12: \( M = 5, S = 3, F = 2, \lambda = 0.8, \alpha = 0.05, \gamma = 0.1 \), and vary the values of \( K \) from 1 to 4.

From Figure 10~15, it appeared that (1) The cost effectiveness maybe insensitive to change in \( \alpha \); (2) The repair rate \( \mu \) increases, the \( C_e \) increases; (3) The values of \( \gamma \) (startup rate) increase, the \( C_e \) decreases; and (4) The number of warm standby (S) decrease then the \( C_e \) increases;

![Figure 10](image1.png) \( C_e \) versus repair rate \( \mu \), with \( M = 5, S = 3, F = 2, K = 2, \lambda = 0.5, \gamma = 0.1 \)

![Figure 11](image2.png) \( C_e \) versus repair rate \( \mu \), with \( M = 5, S = 3, F = 2, K = 2, \lambda = 0.5, \alpha = 0.05 \)

![Figure 12](image3.png) \( C_e \) versus repair rate \( \mu \), with \( M = 5, S = 3, F = 2, K = 2, \lambda = 0.8, \alpha = 0.05, \gamma = 0.1 \)

![Figure 13](image4.png) \( C_e \) versus repair rate \( \mu \), with \( M = 5, F = 2, K = 2, \lambda = 0.8, \alpha = 0.05, \gamma = 0.1 \)
Figure 14. Ce versus repair rate \( \mu \), with \( M = 5, S = 3, K = 2, \lambda = 0.8, \alpha = 0.05, \gamma = 0.1 \)

Figure 15. Ce versus repair rate \( \mu \), with for \( M = 5, S = 3, F = 2, \lambda = 0.8, \alpha = 0.05, \gamma = 0.1 \)

6. CONCLUSIONS

In this paper, we studied an \( F \)-policy M/M/1/K machine repair system and derived the close-form solutions for the steady-state availability, \( A(\infty) \), the steady-state busy period, \( B(\infty) \) and the mean time to system failure, MTTF. From all figures, we conclude that \( \alpha \) rarely affects the profit and the cost effectiveness. The major changes in various performance measures are the failure rate of operating machine \( \lambda \) and the repair rate of server \( \mu \). We also calculated various system performance measures under optimal operating conditions. This research presents an extension of the Markovian model theory and the analysis of the model will provide a useful performance evaluation tool for more general situations arising in practical applications, such as the manufacturing of printed circuit boards and many other related machine repair problems.

7. REFERENCES


Figure 1. State-transition rate diagram for the F-policy M/M/1/N machine repair system.

\[ \lambda_n = \begin{cases} 
M\lambda + (S - n)\alpha & n = 0, 1, \ldots, S \\
(M + S - n)\lambda & n = S + 1, \ldots, L - 1 \\
0 & otherwise
\end{cases} \]