Optimal retailer’s replenishment policy for the EPQ model

under supplier’s trade credit policy

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Abstract

This paper investigates the optimal replenishment policy under conditions of permissible delay in payments within the economic production quantity (EPQ) framework. In 1985, Goyal assumed that:

1. The unit selling price and the unit purchasing price are equal.
2. The replenishment rate is infinite.
3. At the end of the credit period, the account is settled. The retailer starts paying for higher interest charges on the items in stock and returns money of the remaining balance immediately when the items are sold.

The main purpose of this paper wants to modify Goyal’s model to presume that the unit selling price and the unit purchasing price are not necessarily equal to reflect the real-life situations, and assumes the replenishment rate is finite. Furthermore, this paper will propose that at the end of the credit period, the retailer will make a partial payment on total purchasing cost and pay off the remaining balance by loan from the bank. One theorem is obtained to explore the convexity of the total annual relevant cost function. Three theorems are developed to determine the optimal cycle time and the optimal order quantity. Numerical examples are given to illustrate these theorems.

Keywords: EPQ, EOQ, trade credit, permissible delay in payments, inventory
1. Introduction

The timing of cash flows from an asset has a lot to do with desirability of the investment. Investment alternatives that produce the same total returns, but with differing times, are not usually equally attractive. The timing of cash flows of an investment proposal is important because the sooner the money becomes available, the sooner it can be used for other worthwhile purposes. Hence, the arrangement of capital of enterprise is an important issue to enterprise itself. The economic order quantity (EOQ) model assumes that that retailer’s capitals are unrestricting and must pay for the items as soon as the items are received. However, this may not be true. In practice, the supplier will offer the retailer a delay period, that is the trade credit period, in paying for the amount of purchasing cost. Before the end of the trade credit period, the retailer can sell the goods and accumulate revenue and earn interest. A higher interest is charged if the payment is not settled by the end of the trade credit period. Therefore, it makes economic sense for the retailer to delay the settlement of the replenishment account up to the last moment of the permissible period allowed by the supplier. In the real world, the supplier often makes use of this policy to promote his commodities.


Goyal (1985) is well known when the inventory systems under conditions of permissible delay in payments, and implicitly makes the following assumptions:
(1) The unit selling price and the unit purchasing price are assumed to be equal. However, in practice, the unit selling price is not lower than the unit purchasing price in general. Consequently, the viewpoint of Goyal (1985) is debatable sometimes.

(2) The items are obtained from an outside supplier and the entire lot size is delivered at the same time. But we know that the infinite replenishment rate is difficult to reach in general.

(3) At the end of the credit period, the account is settled. The retailer starts paying for higher interest charges on the items in stock and returns money of the remaining balance immediately when the items are sold. What the above statement describes is just one way of the arrangements of capitals of enterprises. Based on considerations of profits, costs and developments of enterprises, enterprises may invest their capitals to the best advantage.

This paper tries to consider some alternatives to move capital to match the policy of enterprise. According to the above arguments, this paper will adopt the following assumptions to modify Goyal’s model.

(i) The unit selling price and the unit purchasing price are not necessarily equal to match the practical situations.

(ii) The replenishment rate is finite to extend the scope of this issue.

(iii) The retailer needs cash for use to business transactions but the retailer does not like to pay higher interest charges too much to the bank. At the end of the credit period, the retailer will make a partial payment on total purchasing cost and pay off the remaining balance by loan from the bank. We also assume the retailer does not return money to the bank until the end of the inventory cycle.

The main purpose of this paper wants to incorporate the above assumptions (i), (ii) and (iii) to develop the retailer’s inventory system to investigate the optimal retailer’s replenishment policies under trade credit policy within the EPQ framework. Then one theorem is obtained to explore the convexity of the total annual relevant cost function. Three theorems are developed to efficiently determine the optimal replenishment policies for the retailer. Finally, numerical examples are given to illustrate the theorems obtained in this paper.

2. Model formulation and convexity

The following notation and assumptions will be used throughout:

Notation:

\[ D = \text{annual demand rate} \]
\[ P = \text{annual replenishment rate, } P > D \]
\[ A = \text{cost of placing one order} \]
\[ \rho = 1 - \frac{D}{P} > 0 \]
\[ c = \text{unit purchasing price per item} \]
\[ s = \text{unit selling price per item of good quality} \]
\[ h = \text{unit stock holding cost per item per year excluding interest charges} \]
\[ I_e = \text{interest which can be earned per $ per year} \]
\[ I_p = \text{interest charges per $ investment in inventory per year} \]
\[ M = \text{the trade credit period} \]
\[ T = \text{the cycle time} \]
\[ T^* = \text{the optimal cycle time} \]
\[ Q^* = \text{the optimal order quantity} = DT^*. \]

**Assumptions:**

1. Demand rate is known and constant.
2. Replenishment rate, \( P \), is known and constant.
3. Shortages are not allowed.
4. Time period is infinite.
5. \( s \geq c \) and \( I_p \geq I_e \).
6. If the credit period is not longer than the cycle length, the retailer can sell the items, accumulate sales revenue and earn interest throughout the inventory cycle. At the end of the credit period, the retailer will make a partial payment \( cDM \) on total purchasing cost to the supplier and pay off the remaining balance by loan from the bank. The retailer does not return money to the bank until the end of the inventory cycle.

**The model**

The total annual relevant cost consists of the following elements.

1. Annual ordering cost = \( \frac{A}{T} \)
2. Annual stock holding cost (excluding interest charges) (shown in Figure 1) =
\[ \frac{hT(P - D)DT}{2T} = \frac{DTh}{2} \left( 1 - \frac{D}{P} \right) = \frac{DTh\rho}{2}. \]
3. There are two cases to occur in cost of interest charges for the items kept in stock per year.
   - Case I : \( M \leq T \), shown in Figure 1.
Interest payable per year = \( cI_p D(T - M)^2 / T \)

Case II : \( T \leq M \)

In this case, no interest charges are paid for the items kept in stock.

(4) There are two cases to occur in interest earned per year.

Case I : \( M \leq T \), shown in Figure 2.

Interest earned per cycle = \( I_s \left[ \int_0^M D(t - M)dt + \frac{1}{M} \int M^T D(t - M)dt + DM(s - c)(T - M) \right] \)

Interest earned per year = \( DI_s \left( \frac{sT}{2} - cM + \frac{cM^2}{T} \right) \)

Case II : \( T \leq M \), shown in Figure 3.

Interest earned per cycle = \( sI_s \left[ \int_0^T D(t - M)dt + DT(M - T) \right] \)

Interest earned per year = \( DsI_s (M - \frac{T}{2}) \)

Combining the above arguments, we obtain that the total annual relevant cost is given by

\[
TVC(T) = \begin{cases} 
A + \frac{DT \rho}{2} + \frac{cI_p D(T - M)^2}{T} - DI_s \left( \frac{sT}{2} - cM + \frac{cM^2}{T} \right) & \text{if } M \leq T \\
A + \frac{DT \rho}{2} - DsI_s (M - \frac{T}{2}) & \text{if } 0 < T \leq M 
\end{cases} 
\]

Let

\[
TVC_1(T) = \frac{A}{T} + \frac{DT \rho}{2} + \frac{cI_p D(T - M)^2}{T} - DI_s \left( \frac{sT}{2} - cM + \frac{cM^2}{T} \right) 
\]

and

\[
TVC_2(T) = \frac{A}{T} + \frac{DT \rho}{2} - DsI_s (M - \frac{T}{2}). 
\]

At \( T=M \), we find \( TVC_1(M)=TVC_2(M) \). Hence \( TVC(T) \) is continuous and well-defined. All \( TVC_1(T) \), \( TVC_2(T) \) and \( TVC(T) \) are defined on \( T > 0 \). Equations (2) and (3) yield

\[
TVC_1'(T) = \frac{-[A + cDM^2(I_p - I_c)]}{T^2} + \frac{D(h \rho + 2cI_p - sI_c)}{2}, 
\]

\[
TVC_1''(T) = \frac{2[A + cDM^2(I_p - I_c)]}{T^3} > 0, 
\]
\[ TVC_2'(T) = \frac{-A}{T^2} + \frac{D(h\rho + sI_c)}{2} \]  \hspace{1cm} (6) \\

and \\

\[ TVC_2''(T) = \frac{2A}{T^3} > 0. \]  \hspace{1cm} (7)

Equations (5) and (7) imply that both \( TVC_1(T) \) and \( TVC_2(T) \) are convex on \( T > 0 \). Then, we can obtain following result.

**Theorem 1:**

1. If \( s \neq c \), then \( TVC_1'(M) \neq TVC_2'(M) \) and \( TVC(T) \) is piecewise convex but not convex.
2. If \( s = c \), then \( TVC_1'(M) = TVC_2'(M) \) and \( TVC(T) \) is convex.

**3. The determination of the optimal cycle time \( T^* \)**

Recall

\[ T_i^* = \sqrt{\frac{2[A + DcM^2(I_p - I_c)]}{D(h\rho + 2cI_p - sI_c)}} \hspace{1cm} \text{if} \hspace{0.2cm} h\rho + 2cI_p - sI_c > 0 \]  \hspace{1cm} (8) \\

and \\

\[ T_2^* = \sqrt{\frac{2A}{D(h\rho + sI_c)}} \]  \hspace{1cm} (9)

introduced in the previous section. Then \( TVC_i'(T_i^*) = 0 \) for all \( i = 1, 2 \). We have the following result.

**Theorem 2 :**

(a) Suppose that \( h\rho + 2cI_p < sI_c \). Then \( T^* = \infty \). (When \( T^* = \infty \), it means that the retailer prefers to keep money and does not return money to the bank.)

(b) Suppose that \( h\rho + 2cI_p = sI_c \). Then

(i) If \( T_2^* \geq M \), then \( T^* = \infty \).

(ii) If \( T_2^* < M \) and \( TVC(T_2^*) \leq -cDM(2I_p - I_c) \), then \( T^* = T_2^* \).

(iii) If \( T_2^* < M \) and \( TVC(T_2^*) > -cDM(2I_p - I_c) \), then \( T^* = \infty \).

**Proof :**

(a) If \( h\rho + 2cI_p < sI_c \), equation (4) implies that \( TVC_i(T) \) is decreasing on \( T > 0 \). Hence
equations 1 (a, b) reveal that \( TVC(T) \) is decreasing on \( T \geq M \). Since
\[
\lim_{T \to \infty} TVC(T) = \lim_{T \to \infty} TVC_1(T) = -cDM(2I_p - I_e) + \lim_{T \to \infty} \frac{DT}{2}(h\rho + 2cI_p - sI_e)
\]
\[
= -\infty
\]
and
\[
\lim_{T \to 0^+} TVC(T) = \lim_{T \to 0^+} TVC_2(T) = \lim_{T \to 0^+} \left[ \frac{A}{T} + \frac{DTh\rho}{2} - DsI_e(M - \frac{T}{2}) \right] = \infty.
\]
Equations (10) and (11) imply \( T^* = \infty \).

(b) If \( h\rho + 2cI_p = sI_e \), equation (4) implies that \( TVC_1(T) \) is decreasing on \( T > 0 \). Hence, equations 1 (a, b) reveal that \( TVC(T) \) is decreasing on \( T \geq M \). Since \( TVC_2'(T^*_2) = 0 \), by the convexity of \( TVC_2(T) \), we see
\[
TVC_2'(T)\begin{cases}
< 0 & \text{if } T < T^*_2 \\
= 0 & \text{if } T = T^*_2 \\
> 0 & \text{if } T > T^*_2
\end{cases}
\]

(i) If \( T^*_2 \geq M \), equations 12 (a, b, c) imply that \( TVC(T) \) is decreasing on \( (0, M] \). Combining the above arguments, we see that \( TVC(T) \) is decreasing on \( T > 0 \). Consequently, \( T^* = \infty \).

(ii) If \( T^*_2 < M \), then \( TVC(T) \) has the minimum value at \( T^*_2 \) on \( (0, M] \). Since
\[
\lim_{T \to 0^+} TVC(T) = \lim_{T \to 0^+} TVC_1(T) = -cDM(2I_p - I_e)
\]
we see that if \( TVC(T^*_2) \leq -cDM(2I_p - I_e) \), then \( T^* = T^*_2 \).

(iii) If \( T^*_2 < M \) and \( TVC(T^*_2) > -cDM(2I_p - I_e) \), equation (13) implies \( T^* = \infty \).

We recall that when \( T^* = \infty \), it means that the retailer prefers to keep money from the remaining balance and does not return money to the bank. If
\[
h\rho + 2cI_p \leq sI_e,
\]
then
\[
s \geq \frac{h\rho + 2cI_p}{I_e}.
\]

In practice, it is possible that equation (14) holds. When a new product is just brought into the
market or oligopoly occurs, equation (14) may happen.

Suppose that an enterprise gets a loan from the bank. Through the appropriate investment of this loan, the enterprise accumulates some money and faces the choice between keeping money versus returning money to the bank as a mode of transaction. Based on consideration of profits, costs and developments of enterprises, enterprises may invest their capitals to the best advantages. Consequently, equation (14) can be treated as a criterion of reference to make a decision about keeping money or returning money to the bank at an appropriate time. Therefore, the reasonable explanation of \( T^* = \infty \) should be that the retailer does not return money to the bank as long as possible when equation (14) holds.

Based on Theorem 2, from now on, we assume \( h \rho + 2cI_p > sI_e \). Consequently, \( T_i^* \) is well-defined. By the convexity of \( TVC_i(T) \) \((i = 1, 2)\), we see

\[
\begin{align*}
TVC_i'(T) &= \begin{cases} 
< 0 & \text{if } T < T_i^* \\
0 & \text{if } T = T_i^* \\
> 0 & \text{if } T > T_i^*
\end{cases} \quad (15a) \\
TVC_i(T) &= \begin{cases} 
< 0 & \text{if } T < T_i^* \\
0 & \text{if } T = T_i^* \\
> 0 & \text{if } T > T_i^*
\end{cases} \quad (15b) \\
\end{align*}
\]

Equations 15 (a, b, c) imply that \( TVC_i(T) \) is decreasing on \((0, T_i^*)\] and increasing on \([T_i^*, \infty)\) for all \( i = 1, 2 \). Equations (4) and (6) yield that

\[
TVC_1'(M) = \frac{-2A + DM^2[h \rho + I_e(2c - s)]}{2M^2} \quad (16)
\]

and

\[
TVC_2'(M) = \frac{-2A + DM^2(h \rho + sI_e)}{2M^2}. \quad (17)
\]

Furthermore, we let

\[
\Delta_1 = -2A + DM^2[h \rho + I_e(2c - s)] \quad (18)
\]

and

\[
\Delta_2 = -2A + DM^2(h \rho + sI_e). \quad (19)
\]

Then, two cases occur:

(I) If \( s > c \), equations (18) and (19) yield that \( \Delta_1 < \Delta_2 \).

(II) If \( s = c \), equations (18) and (19) yield that \( \Delta_1 = \Delta_2 = -2A + DM^2(h \rho + cI_e) = \Delta \).

**Theorem 3** : Suppose that \( s > c \). Then

(A) If \( \Delta_1 \geq 0 \), then \( TVC(T^*) = TVC(T_2^*) \) and \( T^* = T_2^* \).

(B) If \( \Delta_2 \leq 0 \), then \( TVC(T^*) = TVC(T_1^*) \) and \( T^* = T_1^* \).
(C) If $\Delta_1 < 0$ and $\Delta_2 > 0$, then $TVC(T^*)=\min\{TVC(T_1^*),TVC(T_2^*)\}$. Hence $T^*$ is $T_1^*$ or $T_2^*$ associated with the least cost.

Proof:

(A) If $\Delta_1 \geq 0$ then $\Delta_2 > 0$. So $TVC_1'(M) \geq 0$ and $TVC_2'(M) > 0$. Equations 15(a, b, c) imply that

(i) $TVC_1(T)$ is increasing on $[M, \infty)$.

(ii) $TVC_2(T)$ is decreasing on $(0, T_2^*)$ and $TVC_2(T)$ is increasing on $[T_2^*, M]$.

Combining (i), (ii) and equations 1(a, b), we have that $TVC(T)$ is decreasing on $(0, T_2^*)$ and increasing on $[T_2^*, \infty)$. Consequently, $T^* = T_2^*$.

(B) If $\Delta_2 \leq 0$ then $\Delta_1 < 0$. So $TVC_1'(M) < 0$ and $TVC_2'(M) \leq 0$. Equations 15(a, b, c) imply that

(i) $TVC_1(T)$ is decreasing on $(M, T_1^*)$ and $TVC_1(T)$ is increasing on $[T_1^*, \infty)$.

(ii) $TVC_2(T)$ is decreasing on $(0, M)$.

Combining (i), (ii) and equations 1(a, b), we have that $TVC(T)$ is decreasing on $(0, T_1^*)$ and increasing on $[T_1^*, \infty)$. Consequently, $T^* = T_1^*$.

(C) If $\Delta_1 < 0$ and $\Delta_2 > 0$. So $TVC_1'(M) < 0$ and $TVC_2'(M) > 0$. Equations 15(a, b, c) imply that

(i) $TVC_1(T)$ is decreasing on $(M, T_1^*)$ and $TVC_1(T)$ is increasing on $[T_1^*, \infty)$.

(ii) $TVC_2(T)$ is decreasing on $(0, T_2^*)$ and $TVC_2(T)$ is increasing on $[T_2^*, M]$.

Combining (i), (ii) and equations 1(a, b), we have that $TVC(T)$ has the minimum value at $T=T_2^*$ on $(0, M)$ and $TVC(T)$ has the minimum value at $T=T_1^*$ on $[M, \infty)$. Hence $TVC(T^*)=\min\{TVC(T_1^*),TVC(T_2^*)\}$. Consequently, $T^*$ is $T_1^*$ or $T_2^*$ associated with the least cost.

Combining the above arguments, this completes the proof of Theorem 3.

When $s = c$, then $h \rho + 2cI_p > cI_c$. Hence, equations (8) and (9) can be rewritten as

$$T_1^* = \sqrt{\frac{2[A + DcM^2(I_p - I_c)]}{D(h \rho + 2cI_p - cI_c)}}$$

and

$$T_2^* = \sqrt{\frac{2A}{D(h \rho + cI_c)}}.$$  \hspace{1cm} (20)

Furthermore, equations (4) and (6) can be reduced to
\[ TVC'_1(M) = TVC'_2(M) = \frac{-2A + DM^2(hp + cl_2)}{2M^2}. \]  

Equation (22) implies that \( TVC(T) \) is convex if \( s = c \). Then, we have the following result.

**Theorem 4**: Suppose that \( s = c \) and \( \Delta = -2A + DM^2(hp + cl_2) \). Then

(A) If \( \Delta > 0 \), then \( T^* = T_2^* \).

(B) If \( \Delta < 0 \), then \( T^* = T_1^* \).

(C) If \( \Delta = 0 \), then \( T^* = T_1^* = T_2^* = M \).

**Proof**:

(A) If \( \Delta > 0 \), then \( TVC'_1(M) > 0 \) and \( TVC'_2(M) > 0 \). Equations 15(a, b, c) imply that

(i) \( TVC_1(T) \) is increasing on \([M, \infty)\).

(ii) \( TVC_2(T) \) is decreasing on \((0, T_2^*)\) and \( TVC_2(T) \) is increasing on \([T_2^*, M]\).

Combining (i), (ii) and equations 1(a, b), we have that \( TVC(T) \) is decreasing on \((0, T_2^*)\) and increasing on \([T_2^*, \infty)\). Consequently, \( T^* = T_2^* \).

(B) If \( \Delta < 0 \), then \( TVC'_1(M) < 0 \) and \( TVC'_2(M) < 0 \). Equations 15(a, b, c) imply that

(i) \( TVC_1(T) \) is decreasing on \((M, T_1^*)\) and \( TVC_1(T) \) is increasing on \([T_1^*, \infty)\).

(ii) \( TVC_2(T) \) is decreasing on \((0, M)\).

Combining (i), (ii) and equations 1(a, b), we have that \( TVC(T) \) is decreasing on \((0, T_1^*)\) and increasing on \([T_1^*, \infty)\). Consequently, \( T^* = T_1^* \).

(C) If \( \Delta = 0 \), then \( TVC'_1(M) = TVC'_2(M) = 0 \). Since \( TVC_1(M) = TVC_2(M) = TVC(M) \), \( TVC(T) \) has the minimum value at \( M \) and \( T^* = T_1^* = T_2^* = M \).

Combining the above arguments, this completes the proof of Theorem 4.

Theorems 3 and 4 immediately determine the optimal cycle time \( T^* \) after computing the numbers \( \Delta_1, \Delta_2 \) and \( \Delta \). Then, we can calculate optimal order quantity by \( DT^* \). Theorems 3 and 4 are very simple.

**4. Numerical examples**

To illustrate all results obtained in this paper, let us apply the proposed method to solve the following numerical examples. The optimal cycle time and optimal order quantity are summarized in Table 1, Table2 and Table 3, respectively.
Table 1 The optimal cycle time and optimal order quantity using Theorem 2

Let \( P=3000 \text{ units/year} \), \( s=50/\text{unit} \), \( h=5/\text{unit/year} \), \( I_p=0.15/\text{$/year} \), \( I_e=0.1/\text{$/year} \), \( M=0.1 \text{ year} \).

<table>
<thead>
<tr>
<th>( A($/order) )</th>
<th>( D(\text{units/year}) )</th>
<th>( c($/unit) )</th>
<th>( h\rho+2cI_p-sI_e )</th>
<th>Other judgment</th>
<th>( T^*(\text{year}) )</th>
<th>( Q^*(\text{units}) )</th>
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<tbody>
<tr>
<td>10</td>
<td>2000</td>
<td>5</td>
<td>&lt;0</td>
<td>( T_2^*=0.1095 &gt; M=0.1 )</td>
<td>( \infty )</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>1000</td>
<td>5.5555</td>
<td>=0</td>
<td>( T_2^<em>=0.0438 &lt; M=0.1 ) and ( TVC(T_2^</em>)=$ -134.9 &lt; -cDM(2I_p-I_e)=$ -111.1 )</td>
<td>( T_2^*=0.0438 )</td>
<td>43.8</td>
</tr>
<tr>
<td>8</td>
<td>1000</td>
<td>5.5555</td>
<td>=0</td>
<td>( T_2^<em>=0.049 &lt; M=0.1 ) and ( TVC(T_2^</em>)=$ -91.8 &gt; -cDM(2I_p-I_e)=$ -111.1 )</td>
<td>( T_2^*=0.049 )</td>
<td>43.8</td>
</tr>
<tr>
<td>10</td>
<td>1000</td>
<td>5.5555</td>
<td>=0</td>
<td>( T_2^<em>=0.049 &lt; M=0.1 ) and ( TVC(T_2^</em>)=$ -91.8 &gt; -cDM(2I_p-I_e)=$ -111.1 )</td>
<td>( T_2^*=0.049 )</td>
<td>43.8</td>
</tr>
</tbody>
</table>

Table 2 The optimal cycle time and optimal order quantity using Theorem 3

Let \( A=50/\text{order} \), \( P=5000 \text{ units/year} \), \( s=50/\text{unit} \), \( h=5/\text{unit/year} \), \( I_p=0.15/\text{$/year} \), \( I_e=0.1/\text{$/year} \), \( M=0.1 \text{ year} \).

<table>
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<tr>
<th>( D(\text{units/year}) )</th>
<th>( c($/unit)=25 )</th>
<th>( c($/unit)=35 )</th>
<th>( c($/unit)=45 )</th>
</tr>
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<tr>
<td>( \Delta_1 )</td>
<td>( \Delta_2 )</td>
<td>( T^* )</td>
<td>( Q^* )</td>
</tr>
<tr>
<td>1000</td>
<td>&lt;0</td>
<td>&lt;0</td>
<td>0.1387</td>
</tr>
<tr>
<td>2000</td>
<td>&lt;0</td>
<td>&gt;0</td>
<td>0.0791</td>
</tr>
<tr>
<td>3000</td>
<td>&lt;0</td>
<td>&gt;0</td>
<td>0.069</td>
</tr>
</tbody>
</table>

Table 3 The optimal cycle time and optimal order quantity using Theorem 4

Let \( A=65/\text{order} \), \( c=s=50/\text{unit} \), \( h=10/\text{unit/year} \), \( I_p=0.15/\text{$/year} \), \( I_e=0.1/\text{$/year} \), \( M=0.1 \text{ year} \).

<table>
<thead>
<tr>
<th>( D(\text{units/year}) )</th>
<th>( P=4000 \text{ units/year} )</th>
<th>( P=5000 \text{ units/year} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta )</td>
<td>( T^* )</td>
<td>( Q^* )</td>
</tr>
<tr>
<td>1000</td>
<td>&lt;0</td>
<td>0.1014</td>
</tr>
<tr>
<td>2000</td>
<td>&gt;0</td>
<td>0.0806</td>
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<tr>
<td>3000</td>
<td>&gt;0</td>
<td>0.076</td>
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</table>
5. Summary

This paper deals with an inventory model to modify Goyal’s model and assume the finite replenishment rate, then provides very efficient and simple solution procedures to determine the optimal cycle time $T^*$. If $hp + 2cl_p < sl_e$, then $T^* \rightarrow \infty$. This result implies that the retailer ought to lengthen the loan period to the bank as possible. Thus, the retailer can get the most benefit from keeping money. When the new product is just now brought into market or oligopoly occurs, equation (14) may happen. If $hp + 2cl_p = sl_e$, then $T^*$ is $\infty$ or $T_2^*$ associated with the least cost. If $hp + 2cl_p > sl_e$ and $s > c$, the determination of $T^*$ depends on $\Delta_1$ and $\Delta_2$. There are three cases to occur as follows:

(1) If $\Delta_1 \geq 0$, then $T^* = T_2^*$.

(2) If $\Delta_2 \leq 0$, then $T^* = T_1^*$.

(3) If $\Delta_1 < 0$ and $\Delta_2 > 0$, then $T^*$ is $T_1^*$ or $T_2^*$ associated with the least cost.

Moreover, if $s = c$, the determination of $T^*$ depends on $\Delta$. There are three cases to be discussed.

(1) If $\Delta > 0$, then $T^* = T_2^*$.

(2) If $\Delta < 0$, then $T^* = T_1^*$.

(3) If $\Delta = 0$, then $T^* = T_1^* = T_2^* = M$.

Finally, numerical examples are given to illustrate all theorems discussed in this paper. Future study may further incorporate the proposed model into more realistic assumptions, such as probabilistic demand and allowable shortages.
References


Shawky, A. I., and Abou-El-Ata, M. O., 2001, Constrained production lot-size model with
trade-credit policy: ’a comparison geometric programming approach via Lagrange’, *Production Planning & Control*, 12, 654-659.

Figure 1. The total accumulation of interest payable when $M \leq T$

$$I_{\text{max}} = (P-D)(DT/P) = DT \rho$$

Figure 2. The total accumulation of interest earned when $M \leq T$

Figure 3. The total accumulation of interest earned when $T \leq M$