Retailer’s Replenishment Policy Under Supplier Credit Linked to Payment Time

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Abstract

In this paper, we investigate the problem of determining a retailer’s optimal cycle time and the optimal payment policy when supplier offers partially or fully permissible delay in payment linking to payment time. The retailer can obtain the larger fraction of the delay payment permitted by the supplier when the retailer chooses the shorter delayed payment period. The supplier uses this policy to attract retailer to pay the payment as soon as possible to shorten the collection period. Mathematical models have been developed for obtaining the optimal cycle time and optimal payment policy for an item under such conditions. Finally, numerical examples are used to illustrate this result and obtain a lot of managerial insights.

Keywords: Inventory, supplier credit, partially permissible delay in payment

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1. Introduction

In the real world, a supplier will often permit a grace period, widely described as the permissible delay period in the published literature, in settling accounts. During the permissible delay period, the retailer can sell the goods and accumulate revenue and earn interest. A higher interest is charged if the payment is not settled by the end of the permissible delay period. The effect of supplier credit policy on inventory problem has received the attention of many researchers.


The timing of cash flows of an investment proposal is important because the sooner the money becomes available, the sooner it can be used for other worthwhile purposes. Hence, the arrangement of capital of enterprise is an important issue to enterprise itself. Therefore, it makes economic sense for the retailer to delay the settlement of the replenishment account up to the last moment of the permissible delay period allowed by the supplier. From the viewpoint of the supplier, the supplier
would like the retailer to make full payment as soon as possible. Recently, Chang [3], Ouyang et al. [18, 19] and Huang and Chung [14] investigated the inventory policy under cash discount and trade credit. They assumed that the supplier would offer the credit terms mixing cash discount and delay payment permitted to the retailer. The retailer could obtain the cash discount when the payment was paid before cash discount period offered by the supplier. Otherwise, the retailer would make full payment within the permissible delay period.

In this paper, we are suggesting another credit policy that the supplier can adopt to accomplish the desired objective. We assume that the retailer can obtain fully permissible delay in payment if the payment is paid before the period of full delay payment permitted by the supplier. Otherwise, the retailer will just obtain partially permissible delay in payment within the period of partial delay payment permitted by the supplier. For example, the supplier provides 100% delay payment permitted if the payment is made within $M_1$ period, otherwise only $\alpha\%$ ($0 \leq \alpha \leq 100$) delay payment permitted if the payment is made within $M_2$ period ($M_1 < M_2$).

The supplier uses the trade credit policy depending on payment time to attract the retailer to pay the payment as soon as possible to shorten the collection period. Under this condition, we model the retailer’s inventory system as a cost minimization problem. The rest of this paper is organized as follows. In Section 2, we describe the notation and assumptions used throughout this paper. Then, we develop the retailer’s inventory models, and prove the convexity of the inventory annual total relevant cost functions. In Section 3, we derive a closed-form solution to the optimal replenishment cycle, and obtain an ease-to-use theorem to determine the optimal replenishment cycle and optimal payment policy. In Section 4, numerical examples are given to illustrate the result developed in this paper and obtain some managerial phenomena. Finally, we draw the conclusions in Section 5.

2. Model formulation

Following notation and assumptions will be used throughout in this paper.

2.1 Notation:

- $D =$ demand rate per year
- $A =$ cost of placing one order
- $c =$ unit purchasing price
- $h =$ unit stock holding cost per year excluding interest charges
- $\alpha =$ the fraction of the delay payment permitted by the supplier per order, $0 \leq \alpha \leq 1$
- $I_e =$ interest earned per $ per year
- $I_k =$ interest charges per $ investment in inventory per year
$M_1 = \text{the period of full delay payment permitted in years}$

$M_2 = \text{the period of partial delay payment permitted in years, } M_1 < M_2$

$T = \text{the cycle time in years}$

$TRC_{11}(T) = \text{the annual total relevant cost when payment is paid at time } M_1 \text{ and } T \geq M_1$

$TRC_{12}(T) = \text{the annual total relevant cost when payment is paid at time } M_1 \text{ and } T \leq M_1$

$TRC_{11}(T) = \begin{cases} TRC_{11}(T) & \text{if } T \geq M_1 \\ TRC_{12}(T) & \text{if } T \leq M_1 \end{cases}$

$TRC_{21}(T) = \text{the annual total relevant cost when payment is paid at time } M_2 \text{ and } T \geq M_2/(1-\alpha)$

$TRC_{22}(T) = \text{the annual total relevant cost when payment is paid at time } M_2 \text{ and } M_2 \leq T \leq M_2/(1-\alpha)$

$TRC_{23}(T) = \text{the annual total relevant cost when payment is paid at time } M_2 \text{ and } T \leq M_2$

$TRC_2(T) = \begin{cases} TRC_{21}(T) & \text{if } T \geq \frac{M_2}{1-\alpha} \\ TRC_{22}(T) & \text{if } M_2 \leq T \leq \frac{M_2}{1-\alpha} \\ TRC_{23}(T) & \text{if } T \leq M_2 \end{cases}$

$TRC(T) = \text{the annual total relevant cost when } T > 0$

$= \begin{cases} TRC_1(T) & \text{if the payment is paid at time } M_1 \\ TRC_2(T) & \text{if the payment is paid at time } M_2 \end{cases}$

$T_1^* = \text{the optimal cycle time of } TRC_1(T)$

$T_2^* = \text{the optimal cycle time of } TRC_2(T)$

$T^* = \text{the optimal cycle time of } TRC(T)$

$Q^* = \text{the optimal order quantity=DT}^*.$

2.2 Assumptions:

(1) Demand rate is known and constant.

(2) Shortages are not allowed.

(3) Time horizon is infinite.

(4) Replenishments are instantaneous.

(5) $I_k \geq I_e$.

(6) Supplier offers fully delayed payment to the retailer if payment is paid within $M_1$, otherwise just partially delayed payment if payment is paid within $M_2$. 
(7) If payment is paid within $M_1$, when the account is settled the retailer starts paying for the interest charges on the items in stock. If payment is paid behind $M_1$ but within $M_2$, as the order is received, the retailer must make a partial payment $(1-\alpha)cDT$ to the supplier. Then the retailer must pay off the remaining balance $\alpha cDT$ at the end of the partially permissible delay period $M_2$.

(8) During the time the account is not settled, generated sales revenue is deposited in an interest-bearing account.

2.3 The model:

The annual total relevant cost consists of the following elements.

(1) Annual ordering cost = \( \frac{A}{T} \).

(2) Annual stock holding cost (excluding interest charges) = \( \frac{DTh}{2} \).

(3) Cost of interest charges for the items kept in stock per year:

Case 1: Payment is paid at time $M_1$, according to assumption (7), there are two cases in terms of interests charged per year.

Case 1.1: $T \geq M_1$.

Annual interests payable = \( cI_k D(T - M_1)^2 / 2T \).

Case 1.2: $T \leq M_1$.

In this case, annual interests payable = 0.

Case 2: Payment is paid at time $M_2$, according to assumption (7), there are three cases in terms of interests charged per year.

Case 2.1: \( \frac{M_2}{1-\alpha} \leq T \), as shown in Figure 1.

Annual interests payable = \( cI_k (\frac{DT^2}{2} - \alpha DT M_2) / T \).

Case 2.2: \( M_2 \leq T \leq \frac{M_2}{1-\alpha} \), as shown in Figure 2.

Annual interests payable

\[
= cI_k \left[ \frac{(1-\alpha)^2 DT^2}{2} + \frac{D(T-M_2)^2}{2} \right] / T = \frac{cI_k D}{2} \left[ (1-\alpha)^2 T^2 + (T-M_2)^2 \right] / T.
\]

Case 2.3: $T \leq M_2$, as shown in Figure 3.

Annual interests payable = \( cI_k \left[ \frac{(1-\alpha)^2 DT^2}{2} \right] / T \).
(4) Interests earned per year:

**Case 1**: Payment is paid at time $M_1$, according to assumption (8), there are two cases in terms of interests earned per year.

- **Case 1.1** $T \geq M_1$.
  
  Interests earned per year = $cI_e\left(\frac{DM_1^2}{2}\right)/T$.

- **Case 1.2** $T \leq M_1$.
  
  Interests earned per year = $cI_e\left[\frac{DT_1^2}{2} + DT(M_1 - T)/T = cI_eDT(M_1 - \frac{T}{2})/T\right]$.

**Case 2**: Payment is paid at time $M_2$, according to assumption (8), there are two cases in terms of interests earned per year.

- **Case 2.1** $T \geq M_2$.
  
  Interests earned per year = $cI_e\left(\frac{DM_2^2}{2}\right)/T$.

- **Case 2.2** $T \leq M_2$.
  
  Interests earned per year = $cI_e\left[\frac{DT_2^2}{2} + DT(M_2 - T)/T = cI_eDT(M_2 - \frac{T}{2})/T\right]$.

From the above arguments, the annual total relevant cost for the retailer can be expressed as:

Annual total relevant cost = ordering cost + stock-holding cost + interest payable – interest earned.

We show that the annual total relevant cost is given by

**Case 1**: Payment is paid at time $M_1$

\[
TRC_1(T) = \begin{cases} 
TRC_{11}(T) & \text{if } T \geq M_1 \\
TRC_{12}(T) & \text{if } 0 < T \leq M_1 
\end{cases}
\]  

(1a)

(1b)

where

\[
TRC_{11}(T) = \frac{A}{T} + \frac{DTh}{2} + \frac{cI_eD(T - M_1)^2}{2T} - \frac{cI_eDM_1^2}{2T}
\]  

(2)

and

\[
TRC_{12}(T) = \frac{A}{T} + \frac{DTh}{2} - DcI_e(M_1 - \frac{T}{2})
\]  

(3)
At $T=M_1$, we find $TRC_{11}(M_1) = TRC_{12}(M_1)$. Hence $TRC_1(T)$ is continuous and well-defined. All $TRC_{11}(T)$, $TRC_{12}(T)$ and $TRC_1(T)$ are defined on $T > 0$.

**Case 2:** Payment is paid at time $M_2$

\[
TRC_2(T) = \begin{cases} 
TRC_{21}(T) & \text{if } T \geq \frac{M_2}{1-\alpha} \\
TRC_{22}(T) & \text{if } M_2 \leq T \leq \frac{M_2}{1-\alpha} \\
TRC_{23}(T) & \text{if } 0 < T \leq M_2 
\end{cases}
\]

where

\[
TRC_{21}(T) = \frac{A}{T} + \frac{DTh}{2} + cl_1DT\left(\frac{T}{2} - \alpha M_2\right)/T - cI_cDM_2^2/2T,
\]

(5)

\[
TRC_{22}(T) = \frac{A}{T} + \frac{DTh}{2} + cI_cD\left[(1-\alpha)^2T^2 + (T-M_2)^2\right]/2T - cI_cDM_2^2/2T
\]

(6)

and

\[
TRC_{23}(T) = \frac{A}{T} + \frac{DTh}{2} + (1-\alpha)^2cI_cDT^2/2T - cI_cDT(M_2-T)/T.
\]

(7)

Since $TRC_{21}(M_2/(1-\alpha)) = TRC_{22}(M_2/(1-\alpha))$ and $TRC_{23}(M_2) = TVC_{23}(M_2)$, $TRC(T)$ is continuous and well-defined. All $TRC_{21}(T)$, $TRC_{22}(T)$, $TRC_{23}(T)$ and $TRC_2(T)$ are defined on $T > 0$.

**2.4 Optimality conditions:**

From equations (2)-(3) and (5)-(7) yield

\[
TRC_{11}'(T) = \frac{-2A + cDM_1^2(I_k - I_e)}{2T^2} + \frac{D(h + cI_e)}{2},
\]

(8)

\[
TRC_{11}''(T) = \frac{2A + cDM_1^2(I_k - I_e)}{T^3} > 0,
\]

(9)

\[
TRC_{12}'(T) = \frac{-A}{T^2} + \frac{D(h + cI_e)}{2},
\]

(10)

\[
TRC_{12}''(T) = \frac{2A}{T^3} > 0,
\]

(11)

\[
TRC_{21}'(T) = \frac{-2A - cDM_2^2I_e}{2T^2} + \frac{D(h + cI_e)}{2},
\]

(12)

\[
TRC_{21}''(T) = \frac{2A - cDM_2^2I_e}{T^3},
\]

(13)
\[ TRC_{22}^{\prime}(T) = \frac{-[2A + cDM_{2}^2(I_k - I_e)]}{2T^2} + \frac{D\{h + cI_k[1 + (1 - \alpha)^2]\}}{2}, \quad (14) \]

\[ TRC_{22}^{\prime\prime}(T) = \frac{2A + cDM_{2}^2(I_k - I_e)}{T^3} > 0, \quad (15) \]

\[ TRC_{23}^{\prime}(T) = -\frac{A}{T^2} + \frac{D\{h + c[(1 - \alpha)^2 I_k + I_e]\}}{2} \quad (16) \]

and

\[ TRC_{23}^{\prime\prime}(T) = \frac{2A}{T^3} > 0. \quad (17) \]

Equations (9), (11), (15) and (17) imply that all \( TRC_{11}(T), \ TRC_{12}(T), \ TRC_{22}(T) \) and \( TRC_{23}(T) \) are convex on \( T > 0 \). Equation (13) implies \( TRC_{23}(T) \) is convex on \( T > 0 \) if \( 2A - cDM_{2}^2I_e > 0 \).

Furthermore, we have \( TRC_{11}'(M_1) = TRC_{12}'(M_1), \ TRC_{21}'(\frac{M_2}{1 - \alpha}) = TRC_{22}'(\frac{M_2}{1 - \alpha}) \) and

\[ TRC_{22}'(M_2) = TRC_{23}'(M_2). \] Therefore, equations 1(a, b) imply that \( TRC_{11}(T) \) is convex on \( T > 0 \) and equations 4(a, b, c) imply that \( TRC_{22}(T) \) is convex on \( T > 0 \) if \( 2A - cDM_{2}^2I_e > 0 \). Then we can obtain the following results.

**Theorem 1:**

(A) If \( 2A - cDM_{2}^2I_e \leq 0 \), then \( TRC_{22}(T) \) is convex on \( (0, M_2/(1 - \alpha)] \) and concave on \( [M_2/(1 - \alpha), \infty) \).

(B) If \( 2A - cDM_{2}^2I_e > 0 \), then \( TRC_{22}(T) \) is convex on \( (0, \infty) \).

### 3. Decision rule of the optimal cycle time \( T^* \) and optimal payment time

Let \( TRC_{11}'(T_{11}^*) = TRC_{12}'(T_{12}^*) = TRC_{21}'(T_{21}^*) = TRC_{22}'(T_{22}^*) = TRC_{23}'(T_{23}^*) = 0 \). We can obtain

\[ T_{11}^* = \sqrt{\frac{2A + cDM_{1}^2(I_k - I_e)}{D(h + cI_k)}}, \quad (18) \]

\[ T_{12}^* = \sqrt{\frac{2A}{D(h + cI_e)}}, \quad (19) \]

\[ T_{21}^* = \sqrt{\frac{2A - cDM_{2}^2I_e}{D(h + cI_k)}} \text{ if } 2A - cDM_{2}^2I_e > 0, \quad (20) \]
\[ T_{22}^* = \frac{2A + cDM_2^2(I_k - I_e)}{D[h + cI_k[1 + (1 - \alpha)^2]]} \]  
and 
\[ T_{23}^* = \frac{2A}{D[h + c[(1 - \alpha)^2I_k + I_e]]}. \]  

Equation (18) gives the optimal value of \( T^* \) for the case when \( T \geq M_1 \) so that \( T_{11}^* \geq M_1 \). We substitute Equation (18) into \( T_{11}^* \geq M_1 \), then we can obtain that 
\[ T_{11}^* \geq M_1 \] if and only if \(-2A + DM_1^2(h + cI_e) \leq 0\).

Likewise, Equation (19) gives the optimal value of \( T^* \) for the case when \( T \leq M_1 \) so that \( T_{12}^* \leq M_1 \). We substitute Equation (19) into \( T_{12}^* \leq M_1 \), then we can obtain that 
\[ T_{12}^* \leq M_1 \] if and only if \(-2A + DM_1^2(h + cI_e) \geq 0\).

Similar discussion, we can obtain following results:

\[ M_2/(1 - \alpha) \leq T_{21}^* \] if and only if \(-2A - cDM_2^2I_e + D(M_2^2/(1 - \alpha))^2(h + cI_k) \leq 0\).

\[ T_{22}^* \leq M_2/(1 - \alpha) \] if and only if \(-2A - cDM_2^2I_e + D(M_2^2/(1 - \alpha))^2(h + cI_k) \geq 0\) and 
\[ M_2 \leq T_{22}^* \] if and only if \(-2A + DM_2^2(h + c[(1 - \alpha)^2I_k + I_e]) \leq 0\),
\[ T_{23}^* \leq M_2 \] if and only if \(-2A + DM_2^2(h + c[(1 - \alpha)^2I_k + I_e]) \geq 0\).

Furthermore, we let 
\[ \Delta_1 = -2A + DM_1^2(h + cI_e), \]  
\[ \Delta_2 = -(2A - cDM_2^2I_e + D(M_2^2/(1 - \alpha))^2(h + cI_k)) \]  
and 
\[ \Delta_3 = -2A + DM_2^2(h + c[(1 - \alpha)^2I_k + I_e]). \]

Since \( M_1 < M_2 \), we can get \( \Delta_2 \geq \Delta_3 > \Delta_1 \) from equations (23)-(25). Summarized above arguments, the optimal cycle time \( T^* \) and optimal payment policy \((M_1 \text{ or } M_2)\) can be obtained as follows.

**Theorem 2 :**

(A) If \( \Delta_1 \geq 0 \), then \( TRC(T^*) = \min \{ TRC_1(T_{12}^*), TRC_2(T_{23}^*) \} \). Hence \( T^* \) is \( T_{12}^* \) or \( T_{23}^* \), optimal payment time is \( M_1 \) or \( M_2 \) associated with the least cost.

(B) If \( \Delta_1 < 0 \) and \( \Delta_3 > 0 \), then \( TRC(T^*) = \min \{ TRC_1(T_{11}^*), TRC_2(T_{23}^*) \} \). Hence \( T^* \) is \( T_{11}^* \) or \( T_{23}^* \), optimal payment time is \( M_1 \) or \( M_2 \) associated with the least cost.

(C) If \( \Delta_2 > 0 \) and \( \Delta_3 \leq 0 \), then \( TRC(T^*) = \min \{ TRC_1(T_{11}^*), TRC_2(T_{22}^*) \} \). Hence \( T^* \) is \( T_{11}^* \) or
optimal payment time is \( M_1 \) or \( M_2 \) associated with the least cost.

(D) If \( \Delta_2 \leq 0 \), then \( TRC(T^*) = \min \{ TRC_1(T_{11}^*), TRC_2(T_{21}^*) \} \). Hence \( T^* \) is \( T_{11}^* \) or \( T_{21}^* \), optimal payment time is \( M_1 \) or \( M_2 \) associated with the least cost.

Theorem 2 immediately determines the optimal cycle time \( T^* \) and optimal payment policy (\( M_1 \) or \( M_2 \)) after computing the numbers \( \Delta_1, \Delta_2, \) and \( \Delta_3 \). Theorem 2 is an efficient finding-solution procedure.

4. Numerical examples

To illustrate the results, let us apply the proposed method to solve the following numerical examples. For convenience, the values of the parameters are selected randomly. The optimal solutions for different parameters of \( \alpha, M_1 \) and \( c \) are shown in Table 1. The following inferences can be made based on Table 1.

(1). For fixed \( M_1 \) and \( c \), the larger value of \( \alpha \) is, the larger value of the optimal cycle time and the lower value of the annual total relevant cost will be as the optimal payment time is \( M_2 \); however, if the optimal payment time is \( M_1 \), the optimal cycle time is independent of the value of \( \alpha \). This result implies that the retailer will order more quantity to get more capital profits offered by the supplier. In addition, we find that the retailer will choose the shorter delayed payment period to get fully delayed payment when \( \alpha \) is small enough.

(2). For fixed \( \alpha \) and \( c \), the larger the value of \( M_1 \) is, the larger value of the optimal cycle time and the lower value of the annual total relevant cost will be as the optimal payment time is \( M_1 \); however, if the optimal payment time is \( M_2 \), the optimal cycle time is independent of the value of \( M_1 \). This result is similar to the above situation. It implies that the retailer will order more quantity to get more capital profits offered by the supplier.

(3). And last, for fixed \( \alpha \) and \( M_1 \), the larger the value of \( c \) is, the smaller value of the optimal cycle time and the lower value of the annual total relevant cost will be. This result implies that the retailer will order less quantity to take the benefits of the trade credit more frequently.
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5. Conclusions

In this paper, we investigated a retailer’s replenishment and payment policy under permissible delay in payment linked to payment time. The supplier offers the trade credit policy to attract the retailer to buy his/her products. But the supplier also hopes that the retailer can pay the full payment as soon as possible to reduce his/her financing risk. This paper stands a practical and new policy. The supplier can handle the fraction of the delay payment permitted to attract the retailer to pay the payment as soon as possible to shorten the collection period. We developed the retailer’s inventory model and provide a very efficient finding-solution procedure. Theorem 2 effectively determines the optimal cycle time $T^*$ and optimal payment policy after computing the numbers $\Delta_1$, $\Delta_2$, and $\Delta_3$. Finally, numerical examples are used to illustrate this result and obtain a lot of managerial insights: (1) the retailer will order more quantity to get more capital profits offered by the supplier when the supplier offers a larger fraction of the delay payment or a longer fully delayed payment period; (2) the retailer will choose the shorter delayed payment period to get fully delayed payment when the fraction of the delay payment permitted by the supplier is small enough; (3) the retailer will order less quantity to take the benefits of the trade credit more frequently when the unit purchasing price is higher.

A future study will further incorporate the proposed model into more realistic assumptions, such as deterioration items, probabilistic demand, allowable shortages, finite replenishment rate, finite time horizon, time value of money, limited storage capacity and others.
References


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Figure 1. The inventory level and the total amount of interest payable when $\frac{M_2}{1-\alpha} \leq T$

Figure 2. The inventory level and the total amount of interest payable when $M_2 \leq T \leq \frac{M_2}{1-\alpha}$
Figure 3. The inventory level and the total amount of interest payable when $0 < T \leq M_2$