Optimal inventory replenishment policy for the EPQ model under trade credit derived without derivatives

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Abstract

Within the economic production quantity (EPQ) framework, the main purpose of this paper is to deal with Chung and Huang’s model (2003) and extend Chung and Huang’s model (2003) by considering the unit selling price higher than the unit purchasing cost using the algebraic method to determine the optimal inventory replenishment policy for the retailer under trade credit. This paper provides this algebraic approach could be used easily to introduce the basic inventory theories to younger students who lack the knowledge of calculus. In addition, we develop an easy-to-use procedure to find the optimal inventory replenishment policy for the retailer in the extended model developed in this paper. Finally, numerical examples are given to illustrate the result obtained in our extended model.

Keywords: EPQ, EOQ, inventory, trade credit, algebraic

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1. Introduction

The traditional economic order quantity (EOQ) model assumes that the retailer must be paid for the items immediately after receiving the consignment. However, in most business transactions, the supplier will allow a specified credit period to the retailer for payment without penalty to stimulate the demand of their products. Before the end of the trade credit period, the retailer can sell the goods and accumulate revenue and earn interest. A higher interest is charged if the payment is not settled by the end of the trade credit period. Recently, several papers have appeared in the literature that treat inventory problems with varying conditions under the consideration of trade credit. Some of the prominent papers are discussed below.

Goyal (1985) established a single-item inventory model under permissible delay in payments. Chung (1998) developed an alternative approach to determine the economic order quantity under condition of permissible delay in payments. Aggarwal and Jaggi (1995) considered the inventory model with an exponential deterioration rate under the condition of permissible delay in payments. Liao et al. (2000) and Sarker et al. (2000) investigated this topic with inflation. Jamal et al. (1997) and Chang and Dye (2001) extended this issue with allowable shortage. Chang et al. (2001) extended this issue with linear trend demand. Hwang and Shinn (1997) modeled an inventory system for retailer’s pricing and lot sizing policy for exponentially deteriorating products under the condition of permissible delay in payment. Jamal et al. (2000) and Sarker et al. (2000) addressed the optimal payment time under permissible delay in payment with deterioration. Chang et al. (2002) discussed this topic with deteriorating items under time-value of money and finite time horizon. Teng (2002) assumed that the selling price not equal to the purchasing price to modify the Goyal’s model (1985). Shinn and Hwang (2003) determined the retailer’s optimal price and order size simultaneously under the condition of order-size-dependent delay in payments. They assumed that the length of the credit period is a function of the retailer’s order size, and also the demand rate is a function of the selling price. Huang (2003) extended this issue under two levels of trade credit and developed an efficient solution procedure to determine the optimal lot-sizing policy of the retailer.
Huang and Chung (2003) extended Goyal’s model (1985) to cash discount policy for early payment. Abad and Jaggi (2003) developed a joint approach to determine for the seller the optimal unit price and the length of the credit period when end demand is price sensitive. Chung and Huang (2003) investigated this issue within EPQ framework and developed an efficient solution procedure to determine the optimal cycle time for the retailer. Salameh et al. (2003) extended this issue to continuous review inventory model.

In previous all published papers which have been derived using differential calculus to find the optimal solution and the need to prove optimality condition with second-order derivatives. The mathematical methodology is difficult to many younger students who lack the knowledge of calculus. In recent papers, Grubbström and Erdem (1999) and Cardenas-Barrón (2001) showed that the formulae for the EOQ and EPQ with backlogging derived without differential calculus. They mentioned that this approach must be considered as a pedagogical advantage for explaining the EOQ and EPQ concepts to students that lack knowledge of derivatives, simultaneous equations and the procedure to construct and examine the Hessian matrix. This algebraic approach could be used easily to introduce the basic inventory theories to younger students who lack the knowledge of calculus.

Therefore, the main purpose of this paper tries to deal with Chung and Huang’s model (2003) using the algebraic method to determine the optimal inventory replenishment policy for the retailer. Chung and Huang’s model (2003) assumed that the unit selling price and the unit purchasing price are equal. However, as we know, the unit selling price is usually significantly higher than the unit purchasing price in order to obtain profit. Consequently, the viewpoint of Chung and Huang (2003) can be extended. So, we want to amend this assumption to extend Chung and Huang’s model (2003) to reflect the real-life situations. Then, an easy-to-use procedure is developed to efficiently determine the optimal inventory replenishment policy in our extended. Finally, numerical examples are given to illustrate the result obtained in the extended model.
2. Chung and Huang’s model

For convenience, most notation and assumptions similar to Chung and Huang (2003) will be used throughout this paper.

Notation

\[ D = \text{demand rate per year} \]
\[ P = \text{replenishment rate per year, } P \geq D \]
\[ \rho = 1 - \frac{D}{P} \geq 0 \]
\[ A = \text{ordering cost per order} \]
\[ c = \text{unit purchasing price} \]
\[ h = \text{unit stock holding cost per year excluding interest charges} \]
\[ I_e = \text{interest earned per } \$ \text{ per year} \]
\[ I_k = \text{interest charged per } \$ \text{ in stocks per year} \]
\[ M = \text{the trade credit period in years} \]
\[ t_P = \text{the production run time in years } = DT/P \]
\[ T = \text{the cycle time in years} \]
\[ TVC(T) = \text{the annual total relevant cost, which is a function of } T \]
\[ T^* = \text{the optimal cycle time of } TVC(T). \]

Assumptions

(1) Demand rate, \( D \), is known and constant.
(2) Replenishment rate, \( P \), is known and constant.
(3) Shortages are not allowed.
(4) Time horizon is infinite.
(5) \( I_k \geq I_e \).
(6) During the time the account is not settled, generated sales revenue is deposited in an interest-bearing account. When \( T \geq M \), the account is settled at \( T = M \) and we start paying for the
interest charges on the items in stock. When \( T \leq M \), the account is settled at \( T=M \) and we do not need to pay any interest charge.

### 2.1 Algebraic modeling Chung and Huang’s model (2003)

The annual total relevant cost for the retailer can be expressed as:

\[
TVC(T) = \text{ordering cost} + \text{stock-holding cost} + \text{interest payable} - \text{interest earned}.
\]

Based on the above notation and assumptions, Chung and Huang (2003) showed that

\[
TVC(T) = \begin{cases} 
TVC_1(T) & \text{if } T \geq PM/D \\
TVC_2(T) & \text{if } M \leq T \leq PM/D \\
TVC_3(T) & \text{if } 0 < T \leq M 
\end{cases}
\]

(1a) \hspace{2cm} (1b) \hspace{2cm} (1c)

where

\[
TVC_1(T) = \frac{A}{T} + \frac{DT_h \rho}{2} + cI_h \rho \left( \frac{DT^2 - PM^2/2}{2} \right) / T - cI_e \left( \frac{DM^2}{2} \right) / T,
\]

(2)

\[
TVC_2(T) = \frac{A}{T} + \frac{DT_h \rho}{2} + cI_h \left[ \frac{D(T - M)^2}{2} \right] / T - cI_e \left( \frac{DM^2}{2} \right) / T
\]

(3)

and

\[
TVC_3(T) = \frac{A}{T} + \frac{DT_h \rho}{2} - cI_e \left[ \frac{DT^2}{2} + DT(M - T) \right] / T.
\]

(4)

Since \( TVC_1(PM/D) = TVC_2(PM/D) \) and \( TVC_3(M) = TVC_3(M) \), \( TVC(T) \) is continuous and well-defined. All \( TVC_1(T), TVC_2(T), TVC_3(T) \) and \( TVC(T) \) are defined on \( T > 0 \).

Then, we can rewrite

\[
TVC_1(T) = \frac{2A + DM^2c(I_k - I_e) - PM^2cI_k}{2T} + \frac{DT \rho(h + cI_k)}{2T}
\]

\[
= \frac{DT \rho(h + cI_k)}{2T} \left\{ T^2 + \frac{2A + DM^2c(I_k - I_e) - PM^2cI_k}{DT \rho(h + cI_k)} \right\}
\]

\[
= \frac{DT \rho(h + cI_k)}{2T} \left[ T - \sqrt{\frac{2A + DM^2c(I_k - I_e) - PM^2cI_k}{DT \rho(h + cI_k)}} \right]^2
\]

\[
+ \sqrt{DT \rho(h + cI_k)} \left[ 2A + DM^2c(I_k - I_e) - PM^2cI_k \right].
\]

(5)
Equation (5) represents that the minimum of $TVC_1(T)$ is obtained when the quadratic non-negative term, depending on $T$, is made equal to zero. Therefore, the optimum value $T_1^*$ is

$$T_1^* = \sqrt{\frac{2A + DM^2c(I_k - I_e)}{D\rho(h + cI_k)}} \quad \text{if} \quad 2A + DM^2c(I_k - I_e) - PM^2cI_k > 0. \quad (6)$$

Therefore, equation (5) has a minimum value for the optimal value of $T_1^*$ reducing $TVC_1(T)$ to

$$TVC_1(T_1^*) = \left\{ \frac{D\rho(h + cI_k)[2A + DM^2c(I_k - I_e) - PM^2cI_k]}{2} \right\}. \quad (7)$$

Similarly, we can derive $TVC_2(T)$ without derivatives as follows:

$$TVC_2(T) = \frac{2A + DM^2c(I_k - I_e) + DT(h\rho + cI_k)}{2T} - cI_kDM$$

$$= \frac{D(h\rho + cI_k)}{2T} \left\{ T^2 + \frac{2A + DM^2c(I_k - I_e)}{D(h\rho + cI_k)} \right\} - cI_kDM$$

$$= \frac{D(h\rho + cI_k)}{2T} \left\{ T - \sqrt{\frac{2A + DM^2c(I_k - I_e)}{D(h\rho + cI_k)}} \right\}^2$$

$$+ \left\{ \sqrt{D(h\rho + cI_k)[2A + DM^2c(I_k - I_e)]} - cI_kDM \right\}. \quad (8)$$

Equation (8) represents that the minimum of $TVC_2(T)$ is obtained when the quadratic non-negative term, depending on $T$, is made equal to zero. Therefore, the optimum value $T_2^*$ is

$$T_2^* = \sqrt{\frac{2A + DM^2c(I_k - I_e)}{D(h\rho + cI_k)}}. \quad (9)$$

Therefore, equation (8) has a minimum value for the optimal value of $T_2^*$ reducing $TVC_2(T)$ to

$$TVC_2(T_2^*) = \left\{ \frac{D(h\rho + cI_k)[2A + DM^2c(I_k - I_e)] - cI_kDM}{2} \right\}. \quad (10)$$

Likewise, we can derive $TVC_3(T)$ algebraically as follows.

$$TVC_3(T) = \frac{A}{T} + \frac{DT(h\rho + cI_k)}{2} - cI_kDM$$

$$= \frac{D(h\rho + cI_k)}{2T} \left\{ T^2 + \frac{2A}{D(h\rho + cI_k)} \right\} - cI_kDM.$$
Equation (11) represents that the minimum of $TVC_3(T)$ is obtained when the quadratic non-negative term, depending on $T$, is made equal to zero. Therefore, the optimum value $T_3^*$ is

$$T_3^* = \frac{2A}{D(h\rho + cI_e)}.$$

Therefore, equation (11) has a minimum value for the optimal value of $T_3^*$ reducing $TVC_3(T)$ to

$$TVC_3(T_3^*) = \{\sqrt{2AD(h\rho + cI_e) - cI_eDM}\}.$$ (13)

### 2.2 Decision rule of the optimal cycle time $T^*$ in Chung and Huang’s model (2003)

From above section 2.1, equation (6) implies that the optimal value of $T$ for the case of $T \geq PM/D$, that is $T_1^* \geq PM/D$. We substitute equation (6) into $T_1^* \geq PM/D$, then we can obtain that

if and only if $2A \geq \frac{M^2}{D}[P(P-D)h + cI_e(P^2 - D^2) + cI_eD^2].$ (14)

Similarly, equation (9) implies that the optimal value of $T$ for the case of $M \leq T \leq PM/D$, that is $M \leq T_2^* \leq PM/D$. We substitute equation (9) into $M \leq T_2^* \leq PM/D$, then we can obtain that

if and only if $DM^2(h\rho + cI_e) \leq 2A \leq \frac{M^2}{D}[P(P-D)h + cI_e(P^2 - D^2) + cI_eD^2].$ (15)

Finally, equation (12) implies that the optimal value of $T$ for the case of $T \leq M$, that is $T_3^* \leq M$. We substitute equation (12) into $T_3^* \leq M$, then we can obtain that

if and only if $2A \leq DM^2(h\rho + cI_e)$.

From above arguments, we can summarize following results:

**Theorem 1:**

(A) If $2A \geq \frac{M^2}{D}[P(P-D)h + cI_e(P^2 - D^2) + cI_eD^2]$, then $T^* = T_1^*$. 

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(B) If \( DM^2(h \rho + cl_h) \leq 2A \leq \frac{M^2}{D}[P(P - D)h + cI_k(P^2 - D^2) + cl_cD^2] \), then \( T^* = T_2^* \).

(C) If \( 2A \leq DM^2(h \rho + cl_h) \), then \( T^* = T_3^* \).

Furthermore, we let \( \Delta_1 = -2A + \frac{M^2}{D}[P(P - D)h + cI_k(P^2 - D^2) + cl_cD^2] \) and \( \Delta_2 = -2A + DM^2(h \rho + cl_h) \). We can easily obtain \( \Delta_1 \geq \Delta_2 \). Then, we can modify Theorem 1 to following Theorem 2.

**Theorem 2:**

(A) If \( \Delta_2 \geq 0 \), then \( T^* = T_3^* \).

(B) If \( \Delta_1 \geq 0 \) and \( \Delta_2 \leq 0 \), then \( T^* = T_2^* \).

(C) If \( \Delta_1 \leq 0 \), then \( T^* = T_1^* \).

Theorem 2 has been discussed in Theorem 3 in Chung and Huang (2003).

### 3. Extended model

In this section, we want to extend the implicit assumption in Chung and Huang’s model (2003) which assumed the unit selling price and the unit purchasing cost are equal. In this extended model, we want to extend Chung and Huang’s model (2003) by considering the unit selling price higher than the unit purchasing cost. So, we must define the extra notation and modified above assumption (6) in section 2.1 as follows:

**Extra notation:**

- \( s = \) unit selling price, \( s > c \)
- \( TRC(T) = \) the annual total relevant cost in this extended model, which is a function of \( T \)
- \( \bar{T}^* = \) the optimal cycle time of \( TRC(T) \).

We amend above assumption (6) in section 2.1 to assumption (3) in Teng (2002).

During the time the account is not settled, generated sales revenue is deposited in an interest-bearing account. At the end of this period, the retailer pays off all units sold, keeps profits,
and starts paying for the interest charges on the items in stocks.

### 3.1 Algebraic modeling the extended model

Based on the above arguments and easy reading for readers, we depict related figures when the two terms of interest payable and interest earned are calculated. Therefore, this extended model can be expressed as follows:

\[
TRC(T) = \begin{cases} 
TRC_1(T) & \text{if } T \geq PM / D \\
TRC_2(T) & \text{if } M \leq T \leq PM / D \\
TRC_3(T) & \text{if } 0 < T \leq M 
\end{cases} 
\]

(17a) \hspace{2cm} (17b) \hspace{2cm} (17c)

where

\[
TRC_1(T) = \frac{A}{T} + \frac{DTh\rho}{2} + cl_k(\frac{DT^2}{2} - \frac{PM^2}{2})/T - sI_c(\frac{DM^2}{2})/T 
\]

as shown in Figures 1 and 3. (18)

\[
TRC_2(T) = \frac{A}{T} + \frac{DTh\rho}{2} + cI_k[D(T-M)^2]/T - sI_c(\frac{DM^2}{2})/T 
\]

as shown in Figures 2 and 3. (19)

and

\[
TRC_3(T) = \frac{A}{T} + \frac{DTh\rho}{2} - sI_c[\frac{DT^2}{2} + DT(M-T)]/T 
\]

as shown in Figure 4. (20)

[ Insert Figures 1 - 4 here]

Since \( TRC_1(PM / D) = TRC_2(PM / D) \) and \( TRC_3(M) = TRC_3(M) \), \( TRC(T) \) is continuous and well-defined. All \( TRC_1(T), TRC_2(T), TRC_3(T) \) and \( TRC(T) \) are defined on \( T > 0 \).

Then, we can rewrite

\[
TRC_1(T) = \frac{2A + DM^2(cI_k - sI_c) - PM^2cI_k + DTh(h+cI_k)}{2T} 
\]

\[
= \frac{D\rho(h+cI_k)}{2T} \left( T^2 + \frac{2A + DM^2(cI_k - sI_c) - PM^2cI_k}{D\rho(h+cI_k)} \right) 
\]

\[
= \frac{D\rho(h+cI_k)}{2T} \left[ T - \sqrt{\frac{2A + DM^2(cI_k - sI_c) - PM^2cI_k}{D\rho(h+cI_k)}} \right]^2 
\]
Equation (21) represents that the minimum of \( TRC_1(T) \) is obtained when the quadratic non-negative term, depending on \( T \), is made equal to zero. Therefore, the optimum value \( \bar{T}_1^* \) is

\[
\bar{T}_1^* = \sqrt{\frac{2A + DM^2(\frac{cl_k - sI_e}{2}) - PM^2cl_k}{D\rho(h + cI_k)}} \quad \text{if} \quad 2A + DM^2(\frac{cl_k - sI_e}{2}) - PM^2cl_k > 0. \tag{22}
\]

Therefore, equation (21) has a minimum value for the optimal value of \( \bar{T}_1^* \) reducing \( TRC_1(T) \) to

\[
TRC_1(\bar{T}_1^*) = \sqrt{D\rho(h + cI_k)[2A + DM^2(\frac{cl_k - sI_e}{2}) - PM^2cl_k]} \tag{23}
\]

Similarly, we can derive \( TRC_2(T) \) without derivatives as follows.

\[
TRC_2(T) = \frac{2A + DM^2(\frac{cl_k - sI_e}{2}) + DT(h\rho + cI_k)}{2T} - cI_k DM
\]

\[
= \frac{D(h\rho + cI_k)}{2T} \left\{T^2 + \frac{2A + DM^2(\frac{cl_k - sI_e}{2})}{D(h\rho + cI_k)} \right\} - cI_k DM
\]

\[
= \frac{D(h\rho + cI_k)}{2T} \left[T - \sqrt{\frac{2A + DM^2(\frac{cl_k - sI_e}{2})}{D(h\rho + cI_k)}} \right]^2 + \sqrt{D(h\rho + cI_k)[2A + DM^2(\frac{cl_k - sI_e}{2}) - cI_k DM]} \tag{24}
\]

Equation (24) represents that the minimum of \( TRC_2(T) \) is obtained when the quadratic non-negative term, depending on \( T \), is made equal to zero. Therefore, the optimum value \( \bar{T}_2^* \) is

\[
\bar{T}_2^* = \sqrt{\frac{2A + DM^2(\frac{cl_k - sI_e}{2})}{D(h\rho + cI_k)}} \quad \text{if} \quad 2A + DM^2(\frac{cl_k - sI_e}{2}) > 0 \tag{25}
\]

Therefore, equation (24) has a minimum value for the optimal value of \( \bar{T}_2^* \) reducing \( TRC_2(T) \) to

\[
TRC_2(\bar{T}_2^*) = \sqrt{D(h\rho + cI_k)[2A + DM^2(\frac{cl_k - sI_e}{2})] - cI_k DM} \tag{26}
\]

Likewise, we can derive \( TRC_3(T) \) algebraically as follows.

\[
TRC_3(T) = \frac{A}{T} + \frac{DT(h\rho + sI_e)}{2} - sI_e DM
\]
\[
\begin{align*}
D(h\rho + sI_e) & = \frac{2A}{T} \left\{ T^2 + \frac{2A}{D(h\rho + sI_e)} \right\} - sI_eDM \\
& = \frac{D(h\rho + sI_e)}{2T} \left[ T - \sqrt{\frac{2A}{D(h\rho + sI_e)}} \right]^2 + \sqrt{2AD(h\rho + sI_e) - sI_eDM}. 
\end{align*}
\]

Equation (27) represents that the minimum of \(TRC_3(T)\) is obtained when the quadratic non-negative term, depending on \(T\), is made equal to zero. Therefore, the optimum value \(\bar{T}_3^*\) is

\[
\bar{T}_3^* = \sqrt{\frac{2A}{D(h\rho + sI_e)}}. 
\]

Therefore, equation (27) has a minimum value for the optimal value of \(\bar{T}_3^*\) reducing \(TRC_3(T)\) to

\[
TRC_3(\bar{T}_3^*) = \left\{ \sqrt{2AD(h\rho + sI_e) - sI_eDM} \right\}. 
\]

### 3.2 Decision rule of the optimal cycle time \(\bar{T}^*\) in the extended model

From above section 3.1, equation (22) implies that the optimal value of \(T\) for the case of \(T \geq PM/D\), that is \(\bar{T}_1^* \geq PM/D\). We substitute equation (22) into \(\bar{T}_1^* \geq PM/D\), then we can obtain that

if and only if \(2A \geq \frac{M^2}{D} \left[ P(P - D)h + cl_k(P^2 - D^2) + sI_eD^2 \right]. \) (30)

Similarly, equation (25) implies that the optimal value of \(T\) for the case of \(M \leq T \leq PM/D\), that is \(M \leq \bar{T}_2^* \leq PM/D\). We substitute equation (25) into \(M \leq \bar{T}_2^* \leq PM/D\), then we can obtain that

if and only if \(DM^2(h\rho + sI_e) \leq 2A \leq \frac{M^2}{D} \left[ P(P - D)h + cl_k(P^2 - D^2) + sI_eD^2 \right]. \) (31)

Finally, equation (28) implies that the optimal value of \(T\) for the case of \(T \leq M\), that is \(\bar{T}_3^* \leq M\). We substitute equation (28) into \(\bar{T}_3^* \leq M\), then we can obtain that

if and only if \(2A \leq DM^2(h\rho + sI_e)\).

(32)

From above arguments, we can summarize following results:
Theorem 3:

(A) If \( 2A \geq \frac{M^2}{D} [P(P-D)h + cI_k(P^2 - D^2) + sI_eD^2] \), then \( \overline{T}^* = \overline{T}_1^* \).

(B) If \( DM^2(h\rho + sI_e) \leq 2A \leq \frac{M^2}{D} [P(P-D)h + cI_k(P^2 - D^2) + sI_eD^2] \), then \( \overline{T}^* = \overline{T}_2^* \).

(C) If \( 2A \leq DM^2(h\rho + sI_e) \), then \( \overline{T}^* = \overline{T}_3^* \).

Furthermore, we let \( \Delta_3 = -2A + \frac{M^2}{D} [P(P-D)h + cI_k(P^2 - D^2) + sI_eD^2] \) and \( \Delta_4 = -2A + DM^2(h\rho + sI_e) \). We can easily obtain \( \Delta_3 \geq \Delta_4 \). Then, we can modify Theorem 3 to following Theorem 4.

Theorem 4:

(A) If \( \Delta_4 \geq 0 \), then \( \overline{T}^* = \overline{T}_3^* \).

(B) If \( \Delta_3 \geq 0 \) and \( \Delta_4 \leq 0 \), then \( \overline{T}^* = \overline{T}_2^* \).

(C) If \( \Delta_3 \leq 0 \), then \( \overline{T}^* = \overline{T}_1^* \).

Theorem 4 is an effective procedure to find the optimal cycle time \( \overline{T}^* \) by easy judgments \( \Delta_3 \) and \( \Delta_4 \). If \( s = c \), we can find \( \Delta_3 = \Delta_1 \) and \( \Delta_4 = \Delta_2 \). Then Theorem 4 will be reduced to Theorem 2. So, Theorem 2 is a special case of Theorem 4. That is, Chung and Huang’s model (2003) is a special case of our extended model. For easy reading, above results are presented in a summary form in the following Table 1.

Table 1 The theoretical results shown in this paper

<table>
<thead>
<tr>
<th>Chung and Huang’s model ((s = c))</th>
<th>Our extended model ((s &gt; c))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta_1 )</td>
<td>( \Delta_2 )</td>
</tr>
<tr>
<td>&gt;0</td>
<td>&gt;0</td>
</tr>
<tr>
<td>&gt;0</td>
<td>&lt;0</td>
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<tr>
<td>&lt;0</td>
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4. Numerical examples

To illustrate the result obtained in our extended model, let us apply the proposed method to efficiently solve the following numerical example.

Table 2 The optimal cycle time with various values of $P$ and $s$

<table>
<thead>
<tr>
<th>$s$/unit</th>
<th>$P=3000$ units/year</th>
<th>$P=4000$ units/year</th>
<th>$P=5000$ units/year</th>
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<td>$\Delta_3$</td>
<td>$\Delta_4$</td>
<td>$\bar{T}^*$</td>
</tr>
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<td>60</td>
<td>$&gt;0$</td>
<td>$&lt;0$</td>
<td>$\bar{T}_2^*=0.118841$</td>
</tr>
<tr>
<td>90</td>
<td>$&gt;0$</td>
<td>$&lt;0$</td>
<td>$\bar{T}_2^*=0.103923$</td>
</tr>
<tr>
<td>120</td>
<td>$&gt;0$</td>
<td>$&gt;0$</td>
<td>$\bar{T}_3^*=0.091971$</td>
</tr>
</tbody>
</table>

To study the effects of replenishment rate per year, $P$, and unit selling price, $s$, on the optimal cycle time for the retailer derived by the proposed method in our extended model, we solve the example on Table 2 with various values of $P$ and $s$. The following inferences can be made based on Table 2. When $P$ is increasing, the optimal cycle time for the retailer are decreasing. So, the retailer will shorten the ordering time interval since the replenishment speed is faster. When $s$ is increasing, the optimal cycle time for the retailer is decreasing. This result implies that the retailer will not order more quantity to enjoy the benefits of the trade credit more frequently when the larger the differences between the unit selling price and the unit purchasing price. This conclusion also had supported by Teng (2002).

5. Conclusions

This paper adopts the algebraic procedure to investigate Chung and Huang’s model (2003) and extend Chung and Huang’s model (2003) by considering the unit selling price higher than the unit purchasing cost to find the optimal cycle time under trade credit within the EPQ framework. Then, we can obtain the optimal cycle time without calculus. This should also mean that this algebraic
approach is a more accessible approach to ease the learning of basic inventory theories for younger students who lack the knowledge of differential calculus. In addition, we develop an easy-to-use procedure to find the optimal inventory replenishment policy for the retailer in our extended model and deduce Chung and Huang’s model (2003) as a special case when the unit selling price and the unit purchasing cost are equal. From the final numerical example, we find the result that the retailer will order less quantity when the replenishment speed is faster. And, the retailer will not order more quantity to enjoy the benefits of the trade credit more frequently when the larger the differences between the unit selling price and the unit purchasing price.

The proposed model can be extended in several ways. For instance, we may generalize the model to allow for shortages, quantity discounts, time value of money, finite time horizon and others.

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References


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Figure 1. The total amount of interest payable when $M < t_P$ (i.e. $PM/D \leq T$)
Figure 2. The total amount of interest payable when $t_P < M \leq T$ (i.e. $M \leq T \leq PM/D$)
Figure 3. The total amount of interest earned when $M \leq T$
Figure 4. The total amount of interest earned when $T \leq M$