Optimal retailer’s inventory policy under two-level trade credit and two-level storage

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Abstract
This paper modifies the assumptions of the classical economic order quantity (EOQ) to two-level trade credit and two-level storage. That is, we assume that the supplier would offer the retailer a delay period and the retailer also adopts the trade credit policy to stimulate his customer demand to develop the retailer’s replenishment model under the retailer’s storage capacity is limited. It is common practice in most inventory systems to hold excess stocks in a rented warehouse (RW) whenever the storage capacity of the owned warehouse (OW) is insufficient. Under these conditions, we model the retailer’s inventory system and three effective theorems are developed to efficiently determine the optimal cycle time for the retailer. Finally, numerical examples are given to illustrate these theorems and the managerial insights from these numerical examples are also obtained.

Keywords: EOQ, inventory, two-level trade credit, two-level storage, permissible delay in payments.

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1. Introduction

The classical economic order quantity (EOQ) model is a useful inventory control model that is widely accepted in industry. The EOQ model implicitly makes following assumption that the retailer’s capitals are unrestricting and the stocks ordered are fully paid for upon receipt. However, this may not be true. In practice, the supplier will offer the retailer a delay period, which is the trade credit period, to encourage retailer to order large quantities. That is one level of trade credit.

Many articles related to one level of trade credit, we describe as follows. Goyal [12] established a single-item inventory model for determining the economic ordering quantity in the case that the supplier offers the retailer the opportunity to pay his account after a fixed time period. Aggarwal and Jaggi [2] extended Goyal’s model [12] to the case of deteriorating items. Chung [7] developed an alternative approach to determine the economic order quantity to modify Goyal’s [12] approach. Jamal et al. [16] further generalized the model to allow for shortages and constant deterioration. Chen and Chuang [6] investigated light buyer’s inventory policy under trade credit by the concept of discounted cash flows. Liao et al. [18] investigated an inventory model for initial-stock-depended demand rate when a delay in payment is permissible. Jamal et al. [17] and Sarkar et al. [20] addressed the optimal payment time under permissible delay in payment with deterioration. Chang and Dye [5] extended the model by Jamal et al. [16] by introducing variable deteriorating rate and backlogging rate inversely proportional to the waiting time. Teng [21] amended Goyal’s model [12] by considering the difference between unit purchase and selling price. Chung and Huang [8] examined this problem within the EPQ framework and developed an efficient procedure to determine the retailer’s optimal ordering policy. Huang and Chung [15] extended Goyal’s model [12] to cash discount policy for early payment. Abad and Jaggi [1] developed a joint approach to determine for the seller the optimal unit price and the length of the credit period when end demand is price sensitive. Chung and Liao [9] dealt with the problem of determining the economic order quantity for exponentially deteriorating items under permissible delay in payments depending on a large order quantity. Huang [14] extended Chung and Huang’s model [8] when the retailer adopts different payment policy and considers the difference between unit purchase and selling price. Recently, Ouyang et al. [19] investigated an inventory model with noninstantaneous receipt under trade credit, in which the supplier provides not only a permissible delay but also a cash discount to the retailer. Chung and Liao [10] incorporated all concepts of a DCF approach and the trade credit linked to ordering quantity and developed a new inventory model for deteriorating items.

All above articles assumed that the supplier would offer the retailer a delay period and the retailer could sell the goods and accumulate revenue and earn interests within the trade credit period. They implicitly assumed that the customer would pay for the items as soon as the items are received from
the retailer. That is, they assumed that the supplier would offer the retailer a delay period but the retailer would not offer the trade credit period to his customer in previously published articles. In most business transactions, this assumption can be extended. Recently, Huang [13] modified this assumption to assume that the retailer will adopt the trade credit policy to stimulate his customer demand to develop the retailer’s replenishment model. That is two-level trade credit. This new viewpoint is more matched real-life situations in the supply chain model. For example, the Toyota Company can require his supplier offered the trade credit to him and he must also offer the trade credit to his dealership. Then, the Toyota Company can offer shorter delay period to his dealership than his supplier offered to him. In this transaction, the Toyota Company can obtain maximum advantages. But Huang’s model [13] investigated the retailer’s inventory system under one level of storage. It implicitly assumed that the retailer’s storage capacity is unlimited. With the high acquisition cost of land observed world-wide, most warehouses are of limited storage capacity. Therefore, this assumption can be extended.

Such the trade credit policy is one kind of encouragement of the retailer to order large quantities because a delay of payments indirectly reduces inventory cost. Hence, the retailer may purchase more goods than that can be stored in his own warehouse (OW). These excess quantities are stored in a rented warehouse (RW). That is two-level storage. In general, the inventory holding charges in RW are higher than those in OW. When the demand occurs, it first is replenished from the RW which storages those exceeding items. This is done to reduce the inventory costs. It is further assumed that the transportation costs between warehouses are negligible. Several researchers have studied in this area such as Benkherouf [3], Bhunia and Maiti [4], Goswami and Chaudhuri [11] and Wu [22].

According to these arguments, this paper wants to develop the retailer’s inventory system as a cost minimization problem under two-level trade credit and two-level storage. Then we develop efficient solution procedures to determine optimal cycle time for the retailer. We deduce some previously published results of other researchers as special cases. Finally, numerical examples are given to illustrate these results obtained in this paper and the managerial insights are drawn.

2. Model formulation and convexity

The following notation and assumptions will be used throughout:

2.1. Notation

\( D = \) annual demand

\( W = \) retailer’s storage capacity
\( A \) = cost of placing one order  
\( c \) = unit purchasing cost  
\( h \) = OW unit stock holding cost per year  
\( k \) = RW unit stock holding cost per year, \((k \geq h)\)  
\( I_e \) = interest which can be earned per $ per year  
\( I_p \) = interest charges per $ investment in inventory per year  
\( M \) = the retailer’s trade credit period offered by supplier in years  
\( N \) = the customer’s trade credit period offered by retailer in years, \( M \geq N \)  
\( T \) = the cycle time in years  
\( t_w \) = the rented warehouse time in years,  
\[
t_w = \begin{cases} 
\frac{D T - W}{D} & \text{if } DT > W \left( T > \frac{W}{D} \right) \\
0 & \text{if } DT \leq W \left( T \leq \frac{W}{D} \right)
\end{cases}
\]
\( TVC(T) \) = the annual total variable cost, which is a function of \( T \)  
\( T^* \) = the optimal cycle time of \( TVC(T) \).

2.2. Assumptions
(1) Demand rate is known and constant.  
(2) Shortages are not allowed.  
(3) Time horizon is infinite.  
(4) Replenishments are instantaneous.  
(5) \( k \geq h \) and \( I_p \geq I_e \).  
(6) If the order quantity is larger than retailer’s storage capacity, the retailer will rent the warehouse to storage these exceeding items. When the demand occurs, it first is replenished from the warehouse which storages those exceeding items.  
(7) When \( T \geq M \), the account is settled at \( T=M \) and the retailer starts paying the interest charges on the items in stock with rate \( I_p \). When \( T \leq M \), the account is settled at \( T=M \) and the retailer does not need to pay any interest charge.  
(8) The retailer can accumulate revenue and earn interest after his customer pays for the amount of purchasing cost to the retailer until the end of the trade credit period offered by the supplier. That is, the retailer can accumulate revenue and earn interest during the period \( N \) to \( M \) with rate \( I_e \) under the condition of trade credit.

The annual total variable cost consists of the following elements. There are three cases to occur:
(1) \( M \geq W/D \geq N \), (2) \( M \geq N \geq W/D \) and (3) \( W/D \geq M \geq N \).

Case I: Suppose that \( M \geq W/D \geq N \).

(1) Annual ordering cost = \( \frac{A}{T} \).

(2) According to assumption (6), annual stock holding cost (excluding interest charges) can be obtained as follows.

(i) : \( W/D < T \).
In this case, the order quantity is larger than retailer’s storage capacity. So the retailer needs to rent the warehouse to store the exceeding items. Hence
Annual stock holding cost = annual stock holding cost of rented warehouse + annual stock holding cost of the storage capacity \( W \)
\[
\frac{kt_w(DT-W)}{2T} + \frac{h[W_t + \frac{W(T-t_w)}{2}]}{T} = k(DT-W)^2 + \frac{hW(2DT-W)}{2DT}.
\]

(ii) : \( T \leq W/D \).
In this case, the order quantity is not larger than retailer’s storage capacity. So the retailer will not necessary to rent warehouse to store items. Hence
Annual stock holding cost = \( \frac{DT h}{2} \).

(3) According to assumptions (7) and (8), capital opportunity cost (interest payable – interest earned) per year can be obtained as follows.

(i) : \( M \leq T \).
Annual capital opportunity cost
\[
= \frac{c l_p D(T-M)^2}{2T} - c l_r [\frac{(DN+DM)(M-N)}{2}] / T = \frac{c l_p D(T-M)^2}{2T} - c l_r D(M^2 - N^2) / 2T.
\]

(ii) : \( N \leq T \leq M \).
Annual capital opportunity cost
\[
= - c l_r [\frac{(DN+DT)(T-N)}{2} + DT(M-T)] / T = - c l_r D(2MT - N^2 - T^2) / 2T.
\]

(iii) : \( 0 < T \leq N \).
Annual capital opportunity cost = \( - c l_r DT(M-N) / T \).

From the above arguments, the annual total variable cost for the retailer can be expressed as
\[
TVC(T) = \text{ordering cost} + \text{stock-holding cost} + \text{capital opportunity cost}
\]
We show that the annual total variable cost, $TVC(T)$, is given by

\[
TVC(T) = \begin{cases} 
TVC_1(T) & \text{if } T \geq M \\
TVC_2(T) & \text{if } \frac{W}{D} < T \leq M \\
TVC_3(T) & \text{if } N \leq T \leq W/D \\
TVC_4(T) & \text{if } 0 < T \leq N 
\end{cases}
\] (1a) \quad (1b) \quad (1c) \quad (1d)

where

\[
TVC_1(T) = \frac{A}{T} + \frac{k(DT - W)^2}{2DT} + \frac{hW(2DT - W)}{2DT} + \frac{cI_pD(T - M)^2}{2T} - \frac{cI_eD(M^2 - N^2)}{2T},
\] (2)

\[
TVC_2(T) = \frac{A}{T} + \frac{k(DT - W)^2}{2DT} + \frac{hW(2DT - W)}{2DT} - \frac{cI_eD(2MT - N^2 - T^2)}{2T},
\] (3)

\[
TVC_3(T) = \frac{A}{T} + \frac{DTh}{2} - \frac{cI_eD(2MT - N^2 - T^2)}{2T},
\] (4)

and

\[
TVC_4(T) = \frac{A}{T} + \frac{DTh}{2} - cI_eD(M - N).
\] (5)

Since $TVC_1(M) = TVC_2(M)$, $TVC_2(W/D) = TVC_3(W/D)$ and $TVC_3(N) = TVC_4(N)$, $TVC(T)$ is continuous and well defined on $T > 0$. All $TVC_1(T)$, $TVC_2(T)$, $TVC_3(T)$, $TVC_4(T)$ and $TVC(T)$ are defined on $T > 0$. Equations (2), (3), (4) and (5) yield

\[
TVC_1'(T) = -\frac{[2A + \frac{W^2}{D}(k - h) + cDM^2(I_p - I_e) + cDN^2I_e]}{2T^2} + \frac{D(k + cI_p)}{2},
\] (6)

\[
TVC_1''(T) = 2A + \frac{W^2}{D}(k - h) + cDM^2(I_p - I_e) + cDN^2I_e > 0,
\] (7)

\[
TVC_2'(T) = -\frac{[2A + \frac{W^2}{D}(k - h) + cDN^2I_e]}{2T^2} + \frac{D(k + cI_e)}{2},
\] (8)

\[
TVC_2''(T) = 2A + \frac{W^2}{D}(k - h) + cDN^2I_e > 0,
\] (9)

\[
TVC_3'(T) = -\frac{2A + cDN^2I_e}{2T^2} + \frac{D(h + cI_e)}{2},
\] (10)

\[
TVC_3''(T) = \frac{2A + cDN^2I_e}{T^3} > 0,
\] (11)

\[
TVC_4'(T) = -\frac{A}{T^2} + \frac{Dh}{2},
\] (12)
and

\[ TVC_4''(T) = \frac{2A}{T^3} > 0, \]  \hspace{1cm} (13)

Equations (7), (9), (11) and (13) imply that \( TVC_1(T) \), \( TVC_2(T) \), \( TVC_3(T) \) and \( TVC_4(T) \) are convex on \( T > 0 \). Moreover, we have \( TVC_1'(M) = TVC_2'(M) \), \( TVC_2'(W/D) = TVC_3'(W/D) \) and \( TVC_3'(N) = TVC_4'(N) \). So \( TVC(T) \) is also convex on \( T > 0 \).

**Case II: Suppose that \( M \geq N \geq W/D \).**

If \( M \geq N \geq W/D \), Equations 1(a, b, c, d) will be modified as

\[
TVC(T) = \begin{cases} 
TVC_1(T) & \text{if } T \geq M \\
TVC_2(T) & \text{if } N \leq T \leq M \\
TVC_3(T) & \text{if } W/D < T \leq N \\
TVC_4(T) & \text{if } 0 < T \leq W/D 
\end{cases} \]  \hspace{1cm} (14a-d)

When \( W/D < T \leq N \), the annual total variable cost, \( TVC_5(T) \), consists of the following elements.

(1) Annual ordering cost = \( \frac{A}{T} \).

(2) In this case, the order quantity is larger than retailer’s storage capacity. So the retailer needs to rent the warehouse to store the exceeding items. Hence

Annual stock holding cost = \( \frac{k(DT - W)^2}{2DT} + \frac{hW(2DT - W)}{2DT} \).

(3) Annual capital opportunity cost = \(-cI_aDT(M - N)/T\).

Combining above elements, we get

\[
TVC_5(T) = \frac{A}{T} + \frac{k(DT - W)^2}{2DT} + \frac{hW(2DT - W)}{2DT} - cI_aD(M - N). \]  \hspace{1cm} (15)

Since \( TVC_1(M) = TVC_2(M) \), \( TVC_2(N) = TVC_3(N) \) and \( TVC_3(W/D) = TVC_4(W/D) \), \( TVC(T) \) is continuous and well defined on \( T > 0 \). All \( TVC_1(T) \), \( TVC_2(T) \), \( TVC_3(T) \), \( TVC_4(T) \) and \( TVC(T) \) are defined on \( T > 0 \). Equation (15) yields

\[
TVC_5'(T) = -\frac{[2A + \frac{W^2}{D}(k - h)]}{2T^2} + \frac{kD}{2} \]  \hspace{1cm} (16)

and

\[
TVC_5''(T) = \frac{2A + \frac{W^2}{D}(k - h)}{T^3} > 0. \]  \hspace{1cm} (17)
Equations (7), (9), (17) and (13) imply that $TVC_1(T)$, $TVC_2(T)$, $TVC_3(T)$ and $TVC_4(T)$ are convex on $T > 0$. Moreover, we have $TVC_1'(M) = TVC_2'(M)$, $TVC_2'(N) = TVC_3'(N)$ and $TVC_3'(W / D) = TVC_4'(W / D)$. So $TVC(T)$ is also convex on $T > 0$.

**Case III: Suppose that $W/D \geq M \geq N$.**

If $W/D \geq M \geq N$, Equations 1(a, b, c, d) will be modified as

\[
TVC(T) = \begin{cases} 
TVC_1(T) & \text{if } T > W/D \\
TVC_6(T) & \text{if } M \leq T \leq W/D \\
TVC_3(T) & \text{if } N \leq T < M \\
TVC_4(T) & \text{if } 0 < T \leq N 
\end{cases}
\]  

When $M \leq T \leq W/D$, the annual total variable cost, $TVC_6(T)$, consists of the following elements.

1. Annual ordering cost $= \frac{A}{T}$.

2. In this case, the order quantity is not larger than retailer’s storage capacity. So the retailer will not necessary to rent warehouse to storage items. Hence

   Annual stock holding cost $= \frac{DTh}{2}$.

3. Annual capital opportunity cost $= \frac{cI_pD(T - M)^2}{2T} - \frac{cI_eD(M^2 - N^2)}{2T}$.

Combining above elements, we get

\[
TVC_6(T) = \frac{A}{T} + \frac{DTh}{2} + \frac{cI_pD(T - M)^2}{2T} - \frac{cI_eD(M^2 - N^2)}{2T}. 
\]  

Since $TVC_1(W/D) = TVC_6(W/D)$, $TVC_6(M) = TVC_3(M)$ and $TVC_3(N) = TVC_4(N)$, $TVC(T)$ is continuous and well defined on $T > 0$. All $TVC_1(T)$, $TVC_6(T)$, $TVC_3(T)$, $TVC_4(T)$ and $TVC(T)$ are defined on $T > 0$. Equation (19) yields

\[
TVC_6'(T) = -\frac{[2A + cDM^2(I_p - I_e) + cDN^2I_e]}{2T^2} + \frac{D(h + cI_p)}{2} 
\]  

and

\[
TVC_6''(T) = \frac{2A + cD[M^2(I_p - I_e) + N^2I_e]}{T^3} > 0. 
\]  

Equations (7), (21), (11) and (13) imply that $TVC_1(T)$, $TVC_6(T)$, $TVC_3(T)$ and $TVC_4(T)$ are convex on $T > 0$. Moreover, we have $TVC_1'(W / D) = TVC_6'(W / D)$, $TVC_1'(M) = TVC_3'(M)$ and
\(TVC_3'(N) = TVC_4'(N). \) So \(TVC(T)\) is also convex on \(T > 0.\)

### 3. Decision rule of the optimal cycle time \(T^*\)

Let \(TVC_i'(T^*) = 0\) for all \(i = 1, 2, 3, 4, 5, 6.\) We can obtain

\[
T_1^* = \sqrt{\frac{2A + \frac{W^2}{D}(k-h) + cDM^2(I_p - I_r) + cDN^2I_e}{D(k + cI_p)}}, \quad (22)
\]

\[
T_2^* = \sqrt{\frac{2A + \frac{W^2}{D}(k-h) + cDN^2I_e}{D(k + cI_e)}}, \quad (23)
\]

\[
T_3^* = \sqrt{\frac{2A + cDN^2I_e}{D(h + cI_e)}}, \quad (24)
\]

\[
T_4^* = \sqrt{\frac{2A}{Dh}}, \quad (25)
\]

\[
T_5^* = \sqrt{\frac{2A + \frac{W^2}{D}(k-h)}{kD}}, \quad (26)
\]

and

\[
T_6^* = \sqrt{\frac{2A + cD[M^2(I_p - I_r) + N^2I_e]}{D(h + cI_p)}}. \quad (27)
\]

Equations (6), (8), (10), (12), (16) and (20) yield that

\[
TVC_1'(M) = TVC_2'(M) = \frac{-2A - \frac{W^2}{D}(k-h) + DM^2(k + cI_e) - cDN^2I_e}{2M^2}, \quad (28)
\]

\[
TVC_2'(W/D) = TVC_3'(W/D) = \frac{-2A + \frac{W^2}{D}(h + cI_e) - cDN^2I_e}{2\left(\frac{W}{D}\right)^2}, \quad (29)
\]

\[
TVC_3'(N) = TVC_4'(N) = \frac{-2A + DN^2h}{2N^2}, \quad (30)
\]

\[
TVC_2'(N) = TVC_5'(N) = \frac{-2A - \frac{W^2}{D}(k-h) + DN^2k}{2N^2}, \quad (31)
\]
\[ TVC'_5(W/D) = TVC'_4(W/D) = \frac{-2A + \frac{W^2}{D}h}{2\left(\frac{W}{D}\right)^2}, \]  \hspace{1cm} (32) \\
\[ TVC'_1(W/D) = TVC'_6(W/D) = \frac{-2A + \frac{W^2}{D}(h + cI_p) - cDM^2(I_p - I_e) - cDN^2I_e}{2\left(\frac{W}{D}\right)^2} \]  \hspace{1cm} (33) \\
and \\
\[ TVC'_6(M) = TVC'_5(M) = \frac{-2A + DM^2(h + cI_e) - cDN^2I_e}{2M^2}. \]  \hspace{1cm} (34) \\

Furthermore, we let \\
\[ \Delta_1 = -2A - \frac{W^2}{D}(k - h) + DM^2(k + cI_e) - cDN^2I_e, \]  \hspace{1cm} (35) \\
\[ \Delta_2 = -2A + \frac{W^2}{D}(h + cI_e) - cDN^2I_e, \]  \hspace{1cm} (36) \\
\[ \Delta_3 = -2A + DN^2h, \]  \hspace{1cm} (37) \\
\[ \Delta_4 = -2A - \frac{W^2}{D}(k - h) + DN^2k, \]  \hspace{1cm} (38) \\
\[ \Delta_5 = -2A + \frac{W^2}{D}h, \]  \hspace{1cm} (39) \\
\[ \Delta_6 = -2A + \frac{W^2}{D}(h + cI_p) - cDM^2(I_p - I_e) - cDN^2I_e, \]  \hspace{1cm} (40) \\
and \\
\[ \Delta_7 = -2A + DM^2(h + cI_e) - cDN^2I_e. \]  \hspace{1cm} (41) \\

3.1 Suppose That \( M \geq W/D \geq N \).

By the convexity of \( TVC_i(T) \) (i = 1, 2, 3, 4), we see

\[ TVC'_i(T) = \begin{cases} 
< 0 & \text{if} \quad T < T_i^* \\
0 & \text{if} \quad T = T_i^* \\
> 0 & \text{if} \quad T > T_i^*. 
\end{cases} \]  \hspace{1cm} (42a, b, c) \\

Equations 42(a, b, c) imply that \( TVC_i(T) \) is decreasing on \((0, T_i^*]\) and increasing on \([T_i^*, \infty)\) for all \( i = 1, 2, 3, 4 \). Equations (35), (36) and (37) imply that \( \Delta_1 \geq \Delta_2 \geq \Delta_3 \). Therefore, the optimal cycle time can be obtained as follows:
Theorem 1: Suppose that $M \geq W/D \geq N$, then

(A) If $\Delta_3 \geq 0$, then $TVC(T^*) = TVC(T_4^*)$ and $T^* = T_4^*$.

(B) If $\Delta_2 \geq 0$ and $\Delta_3 < 0$, then $TVC(T^*) = TVC(T_3^*)$ and $T^* = T_3^*$.

(C) If $\Delta_1 > 0$ and $\Delta_2 < 0$, then $TVC(T^*) = TVC(T_2^*)$ and $T^* = T_2^*$.

(D) If $\Delta_1 \leq 0$, then $TVC(T^*) = TVC(T_1^*)$ and $T^* = T_1^*$.

Proof. See Appendix.

3.2 Suppose That $M \geq N \geq W/D$.

By the convexity of $TVC_i(T)$ ($i = 1, 2, 5, 4$), we see

\[
TVC_i(T) = \begin{cases} 
< 0 & \text{if } T < T_i^* \\
0 & \text{if } T = T_i^* \\
> 0 & \text{if } T > T_i^*. 
\end{cases} \tag{43a}
\]

Equations 43(a, b, c) imply that $TVC_i(T)$ is decreasing on $(0, T_i^*)$ and increasing on $[T_i^*, \infty)$ for all $i = 1, 2, 5, 4$. Equations (35), (38) and (39) imply that $\Delta_i \geq \Delta_4 \geq \Delta_1$. Therefore, the optimal cycle time can be obtained as follows:

Theorem 2: Suppose that $M \geq N \geq W/D$, then

(A) If $\Delta_5 \geq 0$, then $TVC(T^*) = TVC(T_4^*)$ and $T^* = T_4^*$.

(B) If $\Delta_4 \geq 0$ and $\Delta_5 < 0$, then $TVC(T^*) = TVC(T_3^*)$ and $T^* = T_3^*$.

(C) If $\Delta_1 > 0$ and $\Delta_4 < 0$, then $TVC(T^*) = TVC(T_2^*)$ and $T^* = T_2^*$.

(D) If $\Delta_1 \leq 0$, then $TVC(T^*) = TVC(T_1^*)$ and $T^* = T_1^*$.

Proof. The proof procedures are similar to Theorem 1.

3.3 Suppose That $W/D \geq M \geq N$.

By the convexity of $TVC_i(T)$ ($i = 1, 6, 3, 4$), we see

\[
TVC_i(T) = \begin{cases} 
< 0 & \text{if } T < T_i^* \\
0 & \text{if } T = T_i^* \\
> 0 & \text{if } T > T_i^*. 
\end{cases} \tag{44a}
\]

Equations 44(a, b, c) imply that $TVC_i(T)$ is decreasing on $(0, T_i^*)$ and increasing on $[T_i^*, \infty)$ for all $i = 1, 6, 3, 4$. Equations (40), (41) and (37) imply that $\Delta_6 \geq \Delta_7 \geq \Delta_3$. Therefore, the optimal cycle time can be obtained as follows:

Theorem 3: Suppose that $W/D \geq M \geq N$, then
(A) If $\Delta_3 \geq 0$, then $TVC(T^*) = TVC(T_4^*)$ and $T^* = T_4^*$.

(B) If $\Delta_7 \geq 0$ and $\Delta_3 < 0$, then $TVC(T^*) = TVC(T_5^*)$ and $T^* = T_5^*$.

(C) If $\Delta_6 \geq 0$ and $\Delta_7 < 0$, then $TVC(T^*) = TVC(T_6^*)$ and $T^* = T_6^*$.

(D) If $\Delta_6 < 0$, then $TVC(T^*) = TVC(T_1^*)$ and $T^* = T_1^*$.

**Proof.** The proof procedures are similar to Theorem 1.

4. Special cases

(I) Huang’s model

When $k = h$, it means that the RW unit stock holding cost and the OW unit stock holding cost are equal. It implies that the retailer’s storage capacity is unlimited. That is one level of storage. Let

\[
TVC_7(T) = \frac{A}{T} + \frac{DTh}{2} + cI_p D(T - M)^2 / 2T - cI_c D(M^2 - N^2) / 2T,
\]

\[
TVC_8(T) = \frac{A}{T} + \frac{DTh}{2} - cI_p D(2MT - N^2 - T^2) / 2T
\]

and

\[
TVC_9(T) = \frac{A}{T} + \frac{DTh}{2} - cI_c D(M - N).
\]

Equations 1(a, b, c, d), 14(a, b, c, d) and 18(a, b, c, d) will be reduced as follows:

\[
TVC(T) = \begin{cases} 
TVC_7(T) & \text{if } T \geq M \\
TVC_8(T) & \text{if } N \leq T \leq M \\
TVC_9(T) & \text{if } 0 < T \leq N
\end{cases}
\]

Equations 48(a, b, c) will be consistent with Equations 1(a, b, c) in Huang [13], respectively. Hence, Huang [13] will be a special case of this paper.

(II) Goyal’s model

When $N = 0$, it means that the supplier would offer the retailer a delay period but the retailer would not offer the delay period to his customer. That is one level of trade credit. Therefore, when $k = h$ and $N = 0$, let

\[
TVC_{10}(T) = \frac{A}{T} + \frac{DTh}{2} + cI_p [\frac{D(T - M)^2}{2}] / T - cI_c [\frac{DM^2}{2}] / T
\]

and

\[
TVC_{11}(T) = \frac{A}{T} + \frac{DTh}{2} - cI_c [\frac{DT^2}{2} + DT(M - T)] / T.
\]
Equations 1(a, b, c, d), 14(a, b, c, d) and 18(a, b, c, d) will be reduced as follows:

\[
TVC(T) = \begin{cases} 
TVC_{10}(T) & \text{if } M \leq T \\ 
TVC_{11}(T) & \text{if } 0 < T \leq M 
\end{cases} \quad (51a)
\]

Equations 51(a, b) will be consistent with Equations (1) and (4) in Goyal [12], respectively. Hence, Goyal [12] will be a special case of this paper.

5. Numerical examples

Example 1: A toy retailer buys a lot of dolls from a toy manufacturer at \( c=\$50 \) per unit. The manufacturer offers the trade credit period \( M=0.1 \) year to his retailer. However, if the payment is not paid in full by the end of \( M \), then 15\% interest (ie, \( I_p=0.15 \)) is charged on the outstanding amount. Suppose the retailer can sell \( D=2000 \) dolls a year, OW unit holding cost \( h=\$5/\text{unit/year} \) and RW unit holding cost \( k=\$10/\text{unit/year} \), and \( I_e=10\% \) if the retailer deposits his revenue into a stock mutual-fund account when his customer has paid the amount of purchasing cost to him. If the ordering cost \( A=\$20, 60 \) or 100 per order, retailer’s OW storage capacity \( W=50, 150 \) or 250 units, and the customer’s trade credit period offered by the retailer \( N=0.01, 0.03, 0.04, 0.05 \) or 0.07 year, using Theorems 1, 2 and 3, we can easily obtain the optimal cycle times as shown in Table 1.

Example 2: Following example 1, we adopt many same values in example 1 such as \( D=2000 \text{units/year}, c=\$50/\text{unit}, h=\$5/\text{unit/year}, I_p=0.15/\text{/year}, I_e=0.1/\text{/year}, M=0.1 \) year, and adopting \( N=0.07 \) year. If the ordering cost \( A=\$10, 45 \) or 80 per order, retailer’s OW storage capacity \( W=100, 150 \) or 250 units, and RW unit holding cost \( k=5, 10 \) or 15, using Theorems 1, 2 and 3, we can easily obtain the optimal cycle times as shown in Table 2.

From following Table 1 and Table 2, we can observe the optimal cycle time with various parameters of \( A, W, N \) and \( k \), respectively. The following inferences can be made based on Table 1 and Table 2.

1. When retailer’s OW storage capacity \( W \) is increasing, the optimal cycle time for the retailer will not decrease. These results are easily understood and can be found in Table 1 and Table 2.

2. When the customer’s trade credit period offered by the retailer \( N \) is increasing, the optimal cycle time for the retailer will not decrease. Table 1 shows this result.

3. When the RW unit stock holding cost \( k \) is increasing, the optimal cycle time for the retailer will not increase. This result can be found in Table 2.
Table 1 The optimal cycle time with various values of $A$, $W$ and $N$

Example 1: Let $D=2000\text{units/year}$, $c=$$50/\text{unit}$, $k=$$10/\text{unit/year}$, $h=$$5/\text{unit/year}$, $I_p=$$0.15/\text{$/\text{year}$}$, $I_e=$$0.1/\text{$/\text{year}$}$, $M=0.1\text{year}$.

Case(i): $W=50\text{ units (}W/D=0.025\text{year)}$

<table>
<thead>
<tr>
<th>$N\text{ (year)}$</th>
<th>$A=$$20/\text{order}$</th>
<th>$A=$$60/\text{order}$</th>
<th>$A=$$100/\text{order}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta_1$</td>
<td>$\Delta_4$</td>
<td>$\Delta_5$</td>
<td>$T^*$</td>
</tr>
<tr>
<td>0.03</td>
<td>$&gt;0$</td>
<td>$&lt;0$</td>
<td>$&lt;0$</td>
</tr>
<tr>
<td>0.05</td>
<td>$&gt;0$</td>
<td>$&gt;0$</td>
<td>$&lt;0$</td>
</tr>
<tr>
<td>0.07</td>
<td>$&gt;0$</td>
<td>$&gt;0$</td>
<td>$&lt;0$</td>
</tr>
</tbody>
</table>

Case(ii): $W=150\text{ units (}W/D=0.075\text{year)}$

<table>
<thead>
<tr>
<th>$N\text{ (year)}$</th>
<th>$A=$$20/\text{order}$</th>
<th>$A=$$60/\text{order}$</th>
<th>$A=$$100/\text{order}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta_1$</td>
<td>$\Delta_2$</td>
<td>$\Delta_3$</td>
<td>$T^*$</td>
</tr>
<tr>
<td>0.01</td>
<td>$&gt;0$</td>
<td>$&gt;0$</td>
<td>$&lt;0$</td>
</tr>
<tr>
<td>0.04</td>
<td>$&gt;0$</td>
<td>$&gt;0$</td>
<td>$&lt;0$</td>
</tr>
<tr>
<td>0.07</td>
<td>$&gt;0$</td>
<td>$&gt;0$</td>
<td>$&gt;0$</td>
</tr>
</tbody>
</table>

Case(iii): $W=250\text{ units (}W/D=0.125\text{year)}$

<table>
<thead>
<tr>
<th>$N\text{ (year)}$</th>
<th>$A=$$20/\text{order}$</th>
<th>$A=$$60/\text{order}$</th>
<th>$A=$$100/\text{order}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta_6$</td>
<td>$\Delta_7$</td>
<td>$\Delta_3$</td>
<td>$T^*$</td>
</tr>
<tr>
<td>0.01</td>
<td>$&gt;0$</td>
<td>$&gt;0$</td>
<td>$&lt;0$</td>
</tr>
<tr>
<td>0.04</td>
<td>$&gt;0$</td>
<td>$&gt;0$</td>
<td>$&lt;0$</td>
</tr>
<tr>
<td>0.07</td>
<td>$&gt;0$</td>
<td>$&gt;0$</td>
<td>$&gt;0$</td>
</tr>
<tr>
<td>$k$/unit/year</td>
<td>$A$=$10$/order</td>
<td>$A$=$45$/order</td>
<td>$A$=$80$/order</td>
</tr>
<tr>
<td>---------------</td>
<td>----------------</td>
<td>----------------</td>
<td>----------------</td>
</tr>
<tr>
<td>$\Delta_1$</td>
<td>$\Delta_4$</td>
<td>$\Delta_5$</td>
<td>$T^*$</td>
</tr>
<tr>
<td>5</td>
<td>&gt;0</td>
<td>&gt;0</td>
<td>&gt;0</td>
</tr>
<tr>
<td>10</td>
<td>&gt;0</td>
<td>&gt;0</td>
<td>&gt;0</td>
</tr>
<tr>
<td>15</td>
<td>&gt;0</td>
<td>&gt;0</td>
<td>&gt;0</td>
</tr>
</tbody>
</table>

### Case(ii): $W=150$ units ($W/D = 0.075$/year)

<table>
<thead>
<tr>
<th>$k$/unit/year</th>
<th>$A$=$10$/order</th>
<th>$A$=$45$/order</th>
<th>$A$=$80$/order</th>
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<tbody>
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<td>$T^*$</td>
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<td>&gt;0</td>
<td>&gt;0</td>
</tr>
<tr>
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<td>&gt;0</td>
<td>&gt;0</td>
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</tr>
<tr>
<td>15</td>
<td>&gt;0</td>
<td>&gt;0</td>
<td>&gt;0</td>
</tr>
</tbody>
</table>

### Case(iii): $W=250$ units ($W/D = 0.125$/year)

<table>
<thead>
<tr>
<th>$k$/unit/year</th>
<th>$A$=$10$/order</th>
<th>$A$=$45$/order</th>
<th>$A$=$80$/order</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta_6$</td>
<td>$\Delta_7$</td>
<td>$\Delta_3$</td>
<td>$T^*$</td>
</tr>
<tr>
<td>5</td>
<td>&gt;0</td>
<td>&gt;0</td>
<td>&gt;0</td>
</tr>
<tr>
<td>10</td>
<td>&gt;0</td>
<td>&gt;0</td>
<td>&gt;0</td>
</tr>
<tr>
<td>15</td>
<td>&gt;0</td>
<td>&gt;0</td>
<td>&gt;0</td>
</tr>
</tbody>
</table>

### 6. Summary and conclusions

This paper investigates the effect of two-level trade credit policy and two-level storage within the economic order quantity (EOQ) framework. We extend Huang’s model [13] to retailer’s limited storage space to develop the retailer’s inventory model. Then, we obtain three effective and easy Theorems to help the decision-maker to determine the optimal cycle time. Theorem 1 gives the decision rule of the optimal cycle time when $M \geq W/D \geq N$. Theorem 2 does the decision rule of the optimal cycle time when $M \geq N \geq W/D$. However, Theorem 3 gives the decision rule of the optimal cycle time when $W/D \geq M \geq N$. Furthermore, we deduce Huang’s model [13] and Goyal’s model [12] as special cases of this paper. Finally, numerical examples are used to illustrate all results and
we can also obtain a lot of managerial insights: (1) the retailer will order more quantity when the retailer owns larger storage space to store more items; (2) the retailer will order more quantity to get more capital profits offered by the supplier to compensate the capital losses when longer trade credit period offered to his customer; (3) the retailer will order less quantity to avoid renting expensive warehouse to store these exceeding items when the unit stock holding cost of rented warehouse is expensive.

A future study will further incorporate the proposed model into more realistic assumptions, such as deterioration items, probabilistic demand, allowable shortages, finite replenishment rate, finite time horizon, time value of money, multi-level trade credit and multi-level storage and others.

**Acknowledgements**

This article is partly supported by NSC Taiwan, project no. NSC 94-2416-H-324-003, and we also would like to thank the CYUT to finance this article.
Appendix

Proof of Theorem 1

(A) If \( \Delta_3 \geq 0 \), then \( \Delta_1 > 0 \) and \( \Delta_2 \geq 0 \). Then \( TVC'_1 (M) = TVC'_2 (M) > 0 \),

\[
TVC'_2 (W / D) = TVC'_3 (W / D) \geq 0 \quad \text{and} \quad TVC'_3 (N) = TVC'_4 (N) \geq 0.
\]
Equations 42(a, b, c) imply that

(i) \( TVC_1 (T) \) is increasing on \([M, \infty)\).

(ii) \( TVC_2 (T) \) is increasing on \((W/D, M]\).

(iii) \( TVC_3 (T) \) is increasing on \([N, W/D]\).

(iv) \( TVC_4 (T) \) is decreasing on \((0, T_4^*] \) and increasing on \([T_4^*, N] \).

Combining (i), (ii), (iii), (iv) and Equations 1(a, b, c, d), we have that \( TVC (T) \) is decreasing on \((0, T_4^*] \) and increasing on \([T_4^*, \infty] \). Consequently, \( T^* = T_4^* \).

(B) If \( \Delta_2 \geq 0 \) and \( \Delta_3 < 0 \), then \( \Delta_1 > 0 \). Then \( TVC'_1 (M) = TVC'_2 (M) > 0 \),

\[
TVC'_2 (W / D) = TVC'_3 (W / D) \geq 0 \quad \text{and} \quad TVC'_3 (N) = TVC'_4 (N) < 0.
\]
Equations 42(a, b, c) imply that

(i) \( TVC_1 (T) \) is increasing on \([M, \infty)\).

(ii) \( TVC_2 (T) \) is increasing on \((W/D, M]\).

(iii) \( TVC_3 (T) \) is decreasing on \([N, T_3^*] \) and increasing on \([T_3^*, W/D]\).

(iv) \( TVC_4 (T) \) is decreasing on \((0, N]\).

Combining (i), (ii), (iii), (iv) and Equations 1(a, b, c, d), we have that \( TVC (T) \) is decreasing on \((0, T_3^*] \) and increasing on \([T_3^*, \infty] \). Consequently, \( T^* = T_3^* \).

(C) If \( \Delta_1 > 0 \) and \( \Delta_2 < 0 \), then \( \Delta_3 < 0 \). Then \( TVC'_1 (M) = TVC'_2 (M) > 0 \),

\[
TVC'_2 (W / D) = TVC'_3 (W / D) < 0 \quad \text{and} \quad TVC'_3 (N) = TVC'_4 (N) < 0.
\]
Equations 42(a, b, c) imply that

(i) \( TVC_1 (T) \) is increasing on \([M, \infty)\).

(ii) \( TVC_2 (T) \) is decreasing on \((W/D, T_2^*]\) and increasing on \([T_2^*, M]\).

(iii) \( TVC_3 (T) \) is decreasing on \([N, W/D]\).

(iv) \( TVC_4 (T) \) is decreasing on \((0, N]\).

Combining (i), (ii), (iii), (iv) and Equations 1(a, b, c, d), we have that \( TVC (T) \) is decreasing on \((0, T_2^*] \) and increasing on \([T_2^*, \infty] \). Consequently, \( T^* = T_2^* \).

(D) If \( \Delta_1 \leq 0 \), then \( \Delta_2 < 0 \) and \( \Delta_3 < 0 \). Then \( TVC'_1 (M) = TVC'_2 (M) \leq 0 \),
\[ TVC_2'(W / D) = TVC_3'(W / D) < 0 \text{ and } TVC_4'(N) = TVC_4'(N) < 0. \]
Equations 42(a, b, c) imply that

(i) \( TVC_1(T) \) is decreasing on \([M, T_1^*]\) and increasing on \([T_1^*, \infty)\).

(ii) \( TVC_2(T) \) is decreasing on \((W/D, M]\).

(iii) \( TVC_3(T) \) is decreasing on \([N, W/D]\).

(iv) \( TVC_4(T) \) is decreasing on \((0, N]\).

Combining (i), (ii), (iii), (iv) and Equations 1(a, b, c, d), we have that \( TVC(T) \) is decreasing on \((0, T_1^*]\) and increasing on \([T_1^*, \infty)\). Consequently, \( T^* = T_1^* \).
References


15. Huang, YF, Chung, KJ. Optimal replenishment and payment policies in the EOQ model under


