Retailer’s Economic Ordering Quantity under Partial Payments Delay: an Algebraic Approach

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Abstract

The main purpose of this paper is to investigate the optimal retailer’s lot-sizing decisions under partial payments delay within the economic order quantity (EOQ) framework. All previously published models concerned with payments delay assumed that the supplier would offer the retailer fully payments delay. However, in this paper, we assume that the supplier would offer the retailer partial payments delay. That is, the retailer must make a partial payment to the supplier when the order is received. Then the retailer must pay off the remaining balance at the end of the permissible delay period. Furthermore, we adopt the assumption that the retailer’s unit selling price and the purchasing price per unit are not necessarily equal. Under these conditions, we model the retailer’s inventory system and an algebraic approach is provided to find the optimal solution. One theorem is developed to efficiently determine the optimal lot-sizing decisions for the retailer. Finally, numerical examples are given to illustrate the theoretical results and to draw managerial insights.

Keywords: EOQ, Inventory, Partial payments delay, Algebraic approach
1. Introduction

Typically, inventory planning takes into account only data from the operations concerns. Therefore, the interdependencies among the operations, financing and marketing concerns are neglected. In the classical economic order quantity (EOQ) model, it was tacitly assumed that the purchaser must pay for the items purchased in the moment the items are received. However, in practice, the supplier frequently offers the trade credit, i.e., payments delay permitted, to its purchasers. Over the years, a number of researchers have appeared in the literature that treat inventory problems with varying conditions under payments delay intended to link financing, marketing as well as operations concerns. Some of the prominent papers are discussed below.

Goyal [1] established a single-item inventory model under permissible delay in payments. Chung [2] developed an efficient decision procedure to determine the economic order quantity under condition of permissible delay in payments. Teng [3] assumed that the selling price was not equal to the purchasing price to modify Goyal’s model [1]. Chung and Huang [4] investigated this issue within EPQ (economic production quantity) framework and developed an efficient solving procedure to determine the optimal replenishment cycle for the retailer. Huang and Chung [5] investigated the inventory policy under cash discount and trade credit. Huang [6] assumed that the retailer would adopt a similar trade credit policy to stimulate its customers’ demand in developing the retailer’s replenishment model. This viewpoint that there are two levels of trade credit corresponds, more closely to the supply chain models in practice. Chung and Huang [7] adopted alternative payment rules to develop the inventory model and obtain different results. Huang [8] adopted the payment rule discussed in Chung and Huang [7], and, assumed finite replenishment rate, to investigate the buyer’s inventory problem. Chung et al. [9] investigated retailer’s lot-sizing policy under permissible delay in payments depending on the ordering quantity. Huang [10] extended Huang6 to develop retailer’s inventory policy under retailer’s storage space limited. Recently, Huang [11] incorporated Chung and Huang [4] and Huang [6] to investigate retailer’s ordering policy.

However, all above published papers dealing with economic order quantity in the presence of payments delay assumed that the supplier offers the buyer full payments delay. That is, the buyer would obtain 100% payments delay if the supplier offered the trade credit policy. We know that this is just an extreme case. In reality, the supplier can relax this extreme case to offer the retailer partial payments delay. That is, the retailer must make a partial payment to the supplier when the order is received. Then, the retailer must pay off the remaining balances at the end of the permissible delay period. From the viewpoint of supplier’s marketing policy, the supplier can use the fraction of the payments delay to agilely control the effects of stimulating the demands from the retailer. This
viewpoint is a realistic and novel one in this research field, hence, forms the focus of the present study. Therefore, we ignore the effect of deteriorating item; inflation and finite time horizon similar to most previously published articles. In addition, we adopt the assumption that the retailer’s unit selling price and the purchasing price per unit are not necessarily equal. Under these conditions, we model the retailer’s inventory system and an algebraic approach is provided to find the optimal solution. One theorem is developed to efficiently determine the optimal lot-sizing decisions for the retailer. Finally, numerical examples are given to illustrate the theoretical results and to draw managerial insights.

2. Model formulation

Notation:

- \( D \) = demand rate per year
- \( A \) = cost of placing one order
- \( c \) = unit purchasing price
- \( s \) = unit selling price, \( s > c \)
- \( h \) = unit stock holding cost per year excluding interest charges
- \( \alpha \) = the fraction of the total amount owed payable at the time of placing an order, \( 0 < \alpha \leq 1 \)
- \( I_e \) = interest earned per $ per year
- \( I_k \) = interest charges per $ investment in inventory per year
- \( M \) = the length of the payments delay period in years
- \( T \) = the cycle time in years
- \( TRC(T) \) = the annual total relevant cost when \( T > 0 \)
- \( T^* \) = the optimal cycle time of \( TRC(T) \)
- \( Q^* \) = the optimal order quantity = \( DT^* \).

Assumptions:

1. Demand rate is known and constant.
2. Shortages are not allowed.
3. Time horizon is infinite.
4. Replenishments are instantaneous.
5. \( I_k \geq I_e \).
6. As the order is filled, the retailer must make a partial payment, \( \alpha cDT \), to the supplier. Then the retailer must pay off the remaining balances, \( (1-\alpha)cDT \), at the end of the payments delay period.
(7) The retailer pays off all units sold and keeps his/her profits, and accumulates revenue and earns interest.

The annual total relevant cost consists of the following elements.

(1) Annual ordering cost = $\frac{A}{T}$.

(2) Annual stock holding cost (excluding interest charges) = $\frac{DTh}{2}$.

(3) From assumptions (6) and (7), there are three cases in terms of annual opportunity cost of the capital.

**Case 1** ($\frac{M}{\alpha} \leq T$, Fig. 1 and Fig. 2)

Annual opportunity cost of the capital

$$cI_k\left[\frac{DT^2}{2} - \frac{(1 - \alpha)DTM}{T} - sI_e\left(\frac{(1 - \alpha)DM^2}{2}\right)/T\right].$$

**Case 2** ($M \leq T < \frac{M}{\alpha}$, Fig. 3 and Fig. 4)

Annual opportunity cost of the capital

$$cI_k\left[\frac{\alpha^2DT^2}{2} + \frac{D(T - M)^2}{2}\right]/T - sI_e\left[\frac{(1 - \alpha)\alpha^2DT^2}{2} + \frac{[(1 - \alpha)\alpha DT + DM](M - \alpha T)}{2}\right]/T$$

$$= cI_k\left[\frac{\alpha^2DT^2}{2} + \frac{D(T - M)^2}{2}\right]/T - sI_e\left[\frac{DM(M - \alpha^2T)}{2}\right]/T.$$

**Case 3** ($T < M$, Fig. 5 and Fig. 6)

Annual opportunity cost of the capital

$$cI_k\left(\frac{\alpha^2DT^2}{2}\right)/T - sI_e\left[\frac{(1 - \alpha)\alpha^2DT^2}{2} + \frac{[(1 - \alpha)\alpha DT + DT](T - \alpha T)}{2} + DT(M - T)\right]/T$$

$$= cI_k\left(\frac{\alpha^2DT^2}{2}\right)/T - sI_e\left[\frac{M - (1 + \alpha^2)T}{2}\right]/T.$$

From the above arguments, the annual total relevant cost for the retailer can be expressed as:

Annual total relevant cost = ordering cost + stock-holding cost + opportunity cost of the capital.

We show that the annual total relevant cost is given by
\[ TRC(T) = \begin{cases} 
TRC_1(T) & \text{if } T \geq \frac{M}{\alpha} \\
TRC_2(T) & \text{if } M \leq T < \frac{M}{\alpha} \\
TRC_3(T) & \text{if } 0 < T < M 
\end{cases} \] (1a)

(1b)

(1c)

where

\begin{align*}
TRC_1(T) &= \frac{A}{T} + \frac{DTh}{2} + cI_k[\frac{DT^2}{2} - (1 - \alpha)DTM]/T - sI_e \left[ \frac{(1 - \alpha)DM^2}{2} \right]/T, \\
TRC_2(T) &= \frac{A}{T} + \frac{DTh}{2} + cI_k[\frac{\alpha^2 DT^2}{2} + \frac{D(T - M)^2}{2}]/T - sI_e \left[ \frac{DM(M - \alpha^2 T)}{2} \right]/T, \\
\text{and} \\
TRC_3(T) &= \frac{A}{T} + \frac{DTh}{2} + cI_k[\frac{\alpha^2 DT^2}{2}]/T - sI_e DT \left[ M - \frac{(1 + \alpha^2 T)}{2} \right]/T. 
\end{align*}

(2)

(3)

(4)

Since \( TRC_1(M/\alpha) = TRC_2(M/\alpha) \) and \( TRC_2(M) = TRC_3(M) \), \( TRC(T) \) is continuous and well-defined.

All \( TRC_1(T) \), \( TRC_2(T) \), \( TRC_3(T) \) and \( TRC(T) \) are defined on \( T > 0 \).

Then, we can rewrite

\begin{align*}
TRC_1(T) &= \frac{D(h + cl_k)}{2T} \left[ T - \sqrt{\frac{2A - s(1 - \alpha)DM^2I_e}{D(h + cl_k)}} \right]^2 \\
&\quad + \left\{ \sqrt{D(h + cl_k)[2A - s(1 - \alpha)DM^2I_e] - c(1 - \alpha)I_kDM} \right\}. 
\end{align*}

(5)

From equation (5) the minimum of \( TRC_1(T) \) is obtained when the quadratic non-negative term, depending on \( T \), is equal to zero. The optimum value \( T_1^* \) is

\[ T_1^* = \sqrt{\frac{2A - s(1 - \alpha)DM^2I_e}{D(h + cl_k)}} \quad \text{if} \quad 2A - s(1 - \alpha)DM^2I_e > 0. \] (6)

Therefore,

\[ TRC_1(T_1^*) = \left\{ \sqrt{D(h + cl_k)[2A - s(1 - \alpha)DM^2I_e] - c(1 - \alpha)I_kDM} \right\}. \] (7)

Similarly, we can derive \( TRC_2(T) \) without derivatives as follows.

\begin{align*}
TRC_2(T) &= \frac{D[h + cl_k(1 + \alpha^2)]}{2T} \left[ T - \sqrt{\frac{2A + DM^2(cI_k - sI_e)}{D[h + cl_k(1 + \alpha^2)]}} \right]^2 \\
&\quad + \left\{ \sqrt{D[h + cl_k(1 + \alpha^2)][2A + DM^2(cI_k - sI_e)] - DM(2cI_k + \frac{\alpha^2}{2} sI_e)} \right\}. 
\end{align*}

(8)

From equation (8) the minimum of \( TRC_2(T) \) is obtained when the quadratic non-negative term,
depending on \( T \), is equal to zero. The optimum value \( T_2^* \) is

\[
T_2^* = \sqrt{\frac{2A + DM^2(cI_k - sI_v)}{D[h + cI_k (1 + \alpha^2)]}} \quad \text{if} \quad 2A + DM^2(cI_k - sI_v) > 0. \tag{9}
\]

Therefore,

\[
TRC_2(T_2^*) = \left\{ \sqrt{D[h + cI_k (1 + \alpha^2)][2A + DM^2(cI_k - sI_v)]} - DM \left( 2cI_k + \frac{\alpha^2}{2} sI_v \right) \right\}. \tag{10}
\]

Likewise, we can derive \( TRC_3(T) \) algebraically as follows.

\[
TRC_3(T) = \frac{D[h + \alpha^2 cI_k + (1 + \alpha^2)sI_v]}{2T} \left[ T - \sqrt{\frac{2A}{D[h + \alpha^2 cI_k + (1 + \alpha^2)sI_v]}} \right]^2 + \left\{ 2AD[h + \alpha^2 cI_k + (1 + \alpha^2)sI_v] - DMsI_v \right\}. \tag{11}
\]

From equation (11) the minimum of \( TRC_3(T) \) is obtained when the quadratic non-negative term, depending on \( T \), is equal to zero. The optimum value \( T_3^* \) is

\[
T_3^* = \sqrt{\frac{2A}{D[h + \alpha^2 cI_k + s(1 + \alpha^2)sI_v]}}. \tag{12}
\]

Therefore,

\[
TRC_3(T_3^*) = \left\{ 2AD[h + \alpha^2 cI_k + (1 + \alpha^2)sI_v] - DMsI_v \right\}. \tag{13}
\]

3. Decision rule of the optimal cycle time \( T^* \)

Equation (6) gives that the optimal value of \( T^* \) for the case when \( T \geq M/\alpha \) so that \( T_1^* \geq M/\alpha \). We substitute equation (6) into \( T_1^* \geq M/\alpha \), then we can obtain that

\[
M/\alpha \leq T_1^* \quad \text{if and only if} \quad -2A + DM^2 \leq 0.
\]

Similarly, we can obtain the following results:

\[
M \leq T_2^* < M/\alpha \quad \text{if and only if} \quad -2A + DM^2 > 0 \quad \text{and}
\]

\[
\quad \quad \quad \quad \text{if and only if} \quad -2A + DM^2(h + c\alpha^2 I_k + sI_v) \leq 0,
\]

\[
T_3^* < M \quad \text{if and only if} \quad -2A + DM^2(h + c\alpha^2 I_k + s(1 + \alpha^2)sI_v) > 0.
\]

Furthermore, we let

\[
\Delta_1 = -2A + DM^2[s(1 - \alpha)I_v + \frac{h + cI_k}{\alpha^2}], \tag{14}
\]

\[
\Delta_2 = -2A + DM^2(sI_v + \frac{h + cI_k}{\alpha^2}). \tag{15}
\]
\[ \Delta_3 = -2A + DM^2(h + c\alpha^2 I_k + sI_e) \]  
(16)

and

\[ \Delta_4 = -2A + DM^2[h + c\alpha^2 I_k + s(1+\alpha^2)I_e)]. \]  
(17)

From equations (14)-(17), we can easily obtain \( \Delta_2 > \Delta_1, \Delta_2 \geq \Delta_3 \) and \( \Delta_4 > \Delta_3 \). Summarized above arguments, the optimal cycle time \( T^* \) can be obtained as follows.

**Theorem 1:**

(A) If \( \Delta_1 > 0 \) and \( \Delta_3 > 0 \), then \( TRC(T^*) = TRC_3(T^*_3) \). Hence \( T^* \) is \( T^*_3 \).

(B) If \( \Delta_1 > 0 \), \( \Delta_3 \leq 0 \) and \( \Delta_4 > 0 \), then \( TRC(T^*) = \min \{ TRC_2(T^*_2), TRC_3(T^*_3) \} \). Hence \( T^* \) is \( T^*_2 \) or \( T^*_3 \) associated with the least cost.

(C) If \( \Delta_1 > 0 \) and \( \Delta_4 \leq 0 \), then \( TRC(T^*) = TRC_2(T^*_2) \). Hence \( T^* \) is \( T^*_2 \).

(D) If \( \Delta_1 \leq 0 \), \( \Delta_2 > 0 \) and \( \Delta_4 > 0 \), then \( TRC(T^*) = \min \{ TRC_1(T^*_1), TRC_2(T^*_2), TRC_3(T^*_3) \} \). Hence \( T^* \) is \( T^*_1 \) or \( T^*_3 \) associated with the least cost.

(E) If \( \Delta_1 \leq 0 \), \( \Delta_2 > 0 \), \( \Delta_3 \leq 0 \) and \( \Delta_4 > 0 \), then \( TRC(T^*) = \min \{ TRC_1(T^*_1), TRC_2(T^*_2), TRC_3(T^*_3) \} \). Hence \( T^* \) is \( T^*_1 \) or \( T^*_3 \) associated with the least cost.

(F) If \( \Delta_1 \leq 0 \), \( \Delta_2 > 0 \) and \( \Delta_4 \leq 0 \), then \( TRC(T^*) = \min \{ TRC_1(T^*_1), TRC_2(T^*_2) \} \). Hence \( T^* \) is \( T^*_1 \) or \( T^*_2 \) associated with the least cost.

(G) If \( \Delta_2 \leq 0 \), \( \Delta_3 \leq 0 \) and \( \Delta_4 > 0 \), then \( TRC(T^*) = \min \{ TRC_1(T^*_1), TRC_3(T^*_3) \} \). Hence \( T^* \) is \( T^*_1 \) or \( T^*_3 \) associated with the least cost.

(H) If \( \Delta_2 \leq 0 \) and \( \Delta_4 \leq 0 \), then \( TRC(T^*) = TRC_1(T^*_1) \). Hence \( T^* \) is \( T^*_1 \).

4. **Numerical examples**

In this section, we provide the following numerical examples to illustrate the theoretical results contained in Theorem 1 as reported in Section 3. For convenience to extend the industry, we assume that a manufacturing factory to decide the optimal cycle time and optimal order quantity of his components when his components supplier offers partial payments delay similar to above descriptions. The optimal solutions for different parameters of \( 1-\alpha \) (0.1, 0.4, 0.7) and \( s \) (10, 15, 20) are shown in Table 1, and the values of the parameters are selected randomly. For easily explaining the managerial implications, we adopt the parameter \( 1-\alpha \) not \( \alpha \).
Table 1. The optimal cycle time and optimal order quantity with various values of $\alpha$ and $s$

<table>
<thead>
<tr>
<th>$1-\alpha$</th>
<th>$s$</th>
<th>$\Delta_1$</th>
<th>$\Delta_2$</th>
<th>$\Delta_3$</th>
<th>$\Delta_4$</th>
<th>Theorem</th>
<th>$T^*$</th>
<th>$Q^*$</th>
<th>$TRC(T^*)$</th>
</tr>
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<tr>
<td>0.1</td>
<td>10</td>
<td>&lt;0</td>
<td>&lt;0</td>
<td>&lt;0</td>
<td>&lt;0</td>
<td>1-(H)</td>
<td>0.123288</td>
<td>123.3</td>
<td>786.37</td>
</tr>
<tr>
<td>0.1</td>
<td>15</td>
<td>&lt;0</td>
<td>&lt;0</td>
<td>&lt;0</td>
<td>&lt;0</td>
<td>1-(H)</td>
<td>0.122913</td>
<td>122.9</td>
<td>783.94</td>
</tr>
<tr>
<td>0.1</td>
<td>20</td>
<td>&lt;0</td>
<td>&lt;0</td>
<td>&lt;0</td>
<td>&gt;0</td>
<td>1-(E)</td>
<td>0.122537</td>
<td>122.5</td>
<td>781.49</td>
</tr>
<tr>
<td>0.4</td>
<td>10</td>
<td>&gt;0</td>
<td>&gt;0</td>
<td>&lt;0</td>
<td>&lt;0</td>
<td>1-(C)</td>
<td>0.120957</td>
<td>121.0</td>
<td>723.14</td>
</tr>
<tr>
<td>0.4</td>
<td>15</td>
<td>&gt;0</td>
<td>&gt;0</td>
<td>&lt;0</td>
<td>&lt;0</td>
<td>1-(C)</td>
<td>0.117381</td>
<td>117.4</td>
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<td>&gt;0</td>
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<td>&lt;0</td>
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<td>124.6</td>
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<td>637.84</td>
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</table>

Example 1: Let $A=$50/order, $D=1000$ units/year, $c=$10/unit, $h=$5/unit/year, $I_k=$0.15/$/year, $I_e=$0.12/$/year and $M=0.1$ year.

To study the effects of the fraction of the payments delay, $1-\alpha$, and unit selling price, $s$, on the optimal lot-sizing policy for the manufacturing factory derived by the proposed method. The following inferences can be made based on Table 1.

1. For fixed $s$, increasing the value of $1-\alpha$ will result in a decrease in the value of the optimal order quantity and a decrease in the value of the annual total relevant cost. However, as the value of $1-\alpha$ larger enough, the value of the optimal order quantity will become an increase but the value of the annual total relevant cost is still decreasing. This result implies that the manufacturing factory will order a smaller quantity to enjoy the benefits of the partial payments delay more frequently when the components supplier offers a larger fraction of the payments delay. But the manufacturing factory will order a larger quantity since the manufacturing factory can enjoy the greater benefits when the components supplier offers a larger enough fraction of the payments delay.

2. For fixed $\alpha$, increasing the value of $s$ will result in a decrease in the value of the optimal order quantity and a decrease in the value of the annual total relevant cost. This result implies that the manufacturing factory will order a smaller quantity to enjoy the benefits of the partial payments delay more frequently in the presence of an increased unit selling price.

5. Conclusions

The assumption in previously published results that the full payments delay is permitted. We
know 100% payments delay is just an extreme case. This paper amends the assumption of the full payments delay to partial payments delay to generalize this topic to increase the contribution. We adopt the assumption that the supplier would offer the partial payments delay to the retailer to model the retailer’s inventory problem. In addition, we use an alternative algebraic approach to find the optimal solution and establish an easy-to-use theorem to help the retailer to find the optimal lot-sizing policy. Finally, some numerical examples are provided to illustrate the theoretical results, and to obtain the following managerial insights: (1) a higher value of the fraction of the delay payments permitted brights about a smaller order quantity and smaller annual total relevant cost, and a higher enough value of the fraction of the delay payments permitted brights about a larger order quantity and smaller annual total relevant cost; (2) a higher value of unit selling price brights about a smaller order quantity and smaller annual total relevant cost.

From the viewpoint of supplier’s marketing policy, the supplier can use the fraction of the delay payments permitted to control more agilely the effects of stimulating the demand from retailer. As such, the more realistic and flexible marketing policy is valuable to the supplier. The proposed model can be extended in several ways. For instance, we may generalize the model to allow for shortages, deteriorating item, probabilistic demand, time value of money, finite time horizon, finite replenishment rate etc.

Acknowledgements

Authors thank NSC in Taiwan and CYUT partially financed this research, and the project no. was NSC 96-2221-E-324-007-MY3.
References


Fig. 1 The inventory level and the total amount of interest payable when $M/\alpha \leq T$

Fig. 2 The total amount of interest earned when $M/\alpha \leq T$
Fig. 3 The inventory level and the total amount of interest payable when $M \leq T < M/\alpha$

Fig. 4 The total amount of interest earned when $M \leq T < M/\alpha$
Fig. 5 The inventory level and the total amount of interest payable when $0 < T < M$

Fig. 6 The total amount of interest earned when $0 < T < M$