Application Of Closed-Form Solution For Normal Surface Displacements On Impacted Half Space : Quantification Of Impact-Echo Signals

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Abstract
In order to investigate the feasibility of deducing a simulated transfer function based on the Rayleigh wave form in an Impact-Echo signal, the analytical solution for the normal surface displacement due to a heaviside force at the half-space was reviewed and used to compute the surface displacement responses resulted from various types of impulse forces. According to a series of numerical studies on the characteristics of Rayleigh wave form in the surface displacement responses, this paper presents the idea of using Rayleigh wave form as a pseudo force or generating an equivalent half-sine impact force to derive an intentionally scaled transfer function. The effect of using such pseudo and equivalent force functions was discussed in details. In the proposed method, the force amplitude was first estimated from an amplitude curve which is established from numerical simulations using half-sine force functions. The recovery of a simulated transfer function was next achieved via the use of an estimated force amplitude and a selected force function. The proposed procedure also results in steady thickness amplitude when measurements on two concrete plates were taken for various impacts associated with different steel balls and different impact locations. The success in recovering constant thickness amplitudes for plate-like structural members proved that the derivation of simulated transfer function is a useful tool in extending the Impact-Echo test. The quantitative evaluation of the interfacial property of the substrate layer will also benefit from this simulated transfer function.

Keywords : Impact-Echo, transfer function, Rayleigh wave form, normal response on a half-space

1. Introduction
The use of an impact-type force as the energy source of stress waves has been adopted in many NDT methods, such as the well-known Impact-Echo method, Sonic Echo/Impulse Response method, and Spectral Analysis of Surface Waves (SASW) method. Some of these NDT methods consist of analysis procedures in the frequency domain, and thus, either the frequency amplitude spectrum or the transfer function is computed for the recorded responses.

For methods involving analysis of amplitude spectra, for example Impact-Echo method, the
frequency of the dominant peak shown in an amplitude spectrum normally provides the thickness information of the specimen. Thus, in an Impact-Echo analysis, it is often called thickness frequency. Since the amplitude changes with magnitude and duration of the impact, the amplitude itself fails to provide further information, namely the characteristics of the interface. Clearly, amplitudes recorded in different tests cannot be compared with each other unless the force functions are given. In order to distinguish between various reflected interfaces, it is necessary to normalize frequency spectra by an equivalent force function so that the thickness amplitude associated with a free surface at bottom remains constant for a given specimen.

When a concrete plate is placed on a material of smaller acoustic impedance such as soil, epoxy, and wood, the thickness frequency of the plate is the same as the one reflected free on bottom surface. However, the amplitude should be lower since stress waves are also propagated into the substrate layer. Accordingly, the material property under the concrete plate could be quantitatively evaluated from the thickness amplitude. Furthermore, the debonded or partially debonded condition between concrete and substrate material might also be identified. In other words, transfer functions are preferable when dealing with the identification of substrate layers.

In a traditional Impact-Echo test, the forcing function is not recorded. In order to recover the force function due to an impact-type force, we first review the explicit solution by Pekeris (1955). His solution for the surface responses due to a concentrated force applying on the surface of a semi-infinite medium is derived based on a fixed Poisson’s ratio of 0.25. We will extend the solution formulation for the normal response to account for an arbitrary Poisson’s ratio. Next, we will summarize the fundamental characteristics of normal surface responses obtained numerically for various types of force functions and for materials with different Poisson’s ratios. A procedure to reproduce a simulated transfer function, proven to be beneficiary to the Impact-Echo analysis, will be proposed in the end.

2. Theoretical Background

Regarding the impact response on a semi-infinite medium, the analytical solution can normally be derived in time domain through the convolution of force functions with the Green function or in frequency domain by inversely transforming the product between frequency contents of the force functions and the transfer function. However, when the response due to heaviside step function is given, an alternative approach is to numerically evaluate solutions by the method of superposition. Procedures using the integral transform technique to derive the analytical solution of the surface response due to heaviside step force were first proposed by Pekeris (1955) and can be
found in several textbooks, such as Achenbach (1973) and Graff (1975). It is shown in equation (1) the solution form of the problem. The normal surface displacement is expressed as the function of a dimensionless time parameter, \( \tau \), the relative ratio of the elapsed time to the arrival time of shear wave. The time history of responses can be obtained by numerically computing the two integral terms.

\[
\begin{align*}
\text{where the two functions have the integral forms as} \\
F_1(\tau) &= \int_{\sigma}^{\tau} \frac{y \sqrt{y^2 - \sigma^2}}{\sqrt{\tau^2 - y^2} R(y)} \, dy \\
F_2(\tau) &= \text{P} \int_{\sigma}^{\tau} \frac{y \sqrt{y^2 - \sigma^2}}{\sqrt{\tau^2 - y^2} R(y)} \, dy
\end{align*}
\]

with \( R(y) \) the characteristic equation of the Rayleigh wave and the symbol \( \text{P} \) representing the principal-value of the integral.

Pekeris (1955) shows that, by change of variables, the integral solutions in equation (1) can be further reduced to an algebraic form of simpler rational functions. The result is the solution for the special case \( \nu = 0.25 \) and is given as equation (2).

\[
u_z(r,0,t) = w(t) =
\begin{align*}
0, & \quad \tau < 1/\sqrt{3} \\
-\frac{Z}{32 \pi r} \left\{ 6 - \left( \frac{3}{\tau^2 - 4} \right)^{\frac{1}{2}} - \left( \frac{3\sqrt{3} + 5}{4 + \sqrt{3} - \tau^2} \right)^{\frac{1}{2}} + \left( \frac{3\sqrt{3} - 5}{4 - \sqrt{3} - \tau^2} \right)^{\frac{1}{2}} \right\}, & \quad 1/\sqrt{3} < \tau < 1 \\
-\frac{Z}{16 \pi r} \left\{ 6 - \left( \frac{3\sqrt{3} + 5}{4 + \sqrt{3} - \tau^2} \right)^{\frac{1}{2}} \right\}, & \quad 1 < \tau < \gamma \\
-\frac{3Z}{8 \pi r}, & \quad \tau > \gamma,
\end{align*}
\]

with
$r$ is radial distance

$\mu$ is Lame's elastic constant = $G$ (shear modulus)

$Z$ is the amplitude of the Heaviside step force

$t$ is time

$\rho$ is mass density

\[
\gamma = \frac{3 + \sqrt{3}}{2}
\]

\[
\tau = \frac{C_s t}{r}
\]

\[
C_s = \sqrt{\frac{\mu}{\rho}}
\]

Although equation (1) can be numerically evaluated without difficulty, an explicit type solution like equation (2) is more straightforward and may be able to provide a quantitative reference for the Rayleigh wave (R-wave) responses. In equation (2), the solution is separated into four parts by $\tau$ being equal to $1/\sqrt{3}$, 1 and $\gamma$. These three values of $\tau$ stand for the first arrivals of P-wave, S-wave, and R-wave, respectively. It is interesting to note that the solution exhibits a singular point at $\tau = \gamma$, which results in the R-wave dominated phenomenon. As a result, the R-wave portion of the surface response is of particular importance in problems associated with surface responses.

Following the solution procedure proposed by Pekeris and keeping Poisson’s ratio, $\nu$, as a variable in the derivation, a general solution valid for all Poisson’s ratios can be obtained in Pekeris’ original format as follows.

\[
\begin{align*}
\left\{ \begin{array}{ll}
w(\tau) = 0 & , \quad \tau < \sigma \\
w(\tau) = \frac{Z}{32\pi\mu r(\sigma^2 - 1)} \Re \left\{ 4 - \frac{\overline{A}}{(\tau^2 - a)^{1/2}} - \frac{\overline{B}}{(\tau^2 - b)^{1/2}} - \frac{\overline{C}}{(\tau^2 - c)^{1/2}} \right\} & , \sigma < \tau < 1 \\
w(\tau) = \frac{Z}{16\pi\mu r(\sigma^2 - 1)} \left\{ 4 - \frac{\overline{C}}{(\tau^2 - c)^{1/2}} \right\} & , 1 < \tau < \sqrt{c} \\
w(\tau) = \frac{Z}{4\pi\mu r(\sigma^2 - 1)} = ZA_0(r, \mu, \nu) & , \tau > \sqrt{c}
\end{array} \right.
\end{align*}
\]

in which $a$, $b$ and $c$ are the three roots of $y^2$ in equation (4).

\[
R(y) = (2y^2 - 1)^2 - 4y^2(y^2 - \sigma^2)^{1/2}(y^2 - 1)^{1/2}
\]

\[
= 16(\sigma^2 - 1)y^6 + 8(3 - 2\sigma^2)y^4 - 8y^2 + 1
\]

\[
= (y^2 - a)(y^2 - b)(y^2 - c)
\]

with $y = C_s/C_R$ and $\sigma = C_s/C_p = \sqrt{(1 - 2\nu)}/2(1 - \nu)$ . $C_p$, $C_s$ and $C_R$ are the wave velocities of the
P-wave, S-wave and R-wave, respectively. Equation (4) is the characteristic equation of the Rayleigh surface wave with \( a \) and \( c \) being denoted the smallest and largest of the three roots of \( R(y^2) = 0 \). In addition, \( A_0 \) is the steady-state displacement due to Heaviside step force and can be defined as a nominal amplitude factor to quantify displacement amplitude caused by impulse-type forces.

Equation (4) indicates that when \( \nu \leq 0.263 \), we have \( a < b < \sigma^2 < 1 < c \) with \( a, b, \) and \( c \) all real numbers. On the other hand, when \( \nu > 0.263 \), \( a \) and \( b \) become a pair of conjugate complex numbers and we have \( \text{Re}(a) = \text{Re}(b) < \sigma^2 < 1 < c \). The remaining three coefficients in the numerators, \( \bar{A}, \bar{B} \), and \( \bar{C} \), are functions of \( a, b, c, \) and \( \sigma^2 \) as shown in equation (5).

\[
\bar{A} = \frac{4a(1-a)^{1/2}(\sigma^2-a)}{(a-b)(a-c)} = \frac{(2a-1)^2(\sigma^2-a)^{1/2}}{(a-b)(a-c)}
\]

\[
\bar{B} = \frac{4b(1-b)^{1/2}(\sigma^2-b)}{(b-c)(b-a)}
\]

\[
\bar{C} = \frac{4c(c-1)^{1/2}(c-\sigma^2)}{(c-a)(c-b)}
\]

When the Poisson’s ratio is 0.25, the three roots of equation (4) are \( a = \frac{1}{4}, b = \frac{3+\sqrt{15}}{4}, c = \frac{3+\sqrt{15}}{4} \) and the constant \( \sigma^2 \) equals 1/3. Substitution of these values into equation (5) gives \( \bar{A} = \frac{2}{3\sqrt{3}}, \bar{B} = -\frac{2\sqrt{3}}{3}, \bar{C} = \frac{2\sqrt{3}}{3}, \) and \( A_0 = \frac{3}{8\sqrt{3}} \cdot \). Thus, equation (3) is reduced to equation (2). Figure 1 concludes the dimensionless displacement responses associated with five Poisson’s ratios, 0.1, 0.18, 0.25, 0.33, and 0.4.

Fig. 1: Vertical displacement at free surface according to equation (3).
For a given force-time function, \( f(t) \), the corresponding vertical displacement at the free surface \( W(t) \) can be computed by employing the method of superposition as shown in Wu et al. (1995). In their work, the force-time function is first discretized into \( n \) segments and expressed as the summation of \( n+1 \) Heaviside functions, \( H(t) \), as shown in equation (6).

\[
f(t) = \sum_{k=0}^{n} a_k H(t - k\Delta t)
\]  

(6)

Then the surface response with respect to any arbitrary force–time function can be evaluated from summing up contributions from all Heaviside functions as equation (7).

\[
W(t) = \sum_{k=0}^{n} a_k w(\tau - k\Delta \tau)
\]  

(7)

where \( w(\tau) \) is shown in equation (3) and the dimensionless time variable \( \tau = tC_s/r \).

3. Numerical Simulations of Impact Responses

In the following, the characteristics of impacted surface responses due to different impact force functions are discussed. Figure 2 shows the typical surface response due to an impact force, in which \( A_t \) and \( A_{\text{peak}} \) denote the amplitudes associated with the trough prior to the R-wave arrival and the compressive (positive) peak of the R-wave signal, respectively. To estimate the duration of the impact, two durations \( \tau_d \) and \( \tau_0 \) are defined in which the former represents the elapsed time between trough and ending of the R-wave signal, and the latter stands for the duration of the entire compressive response between two zero crossings.

![Fig. 2: A typical surface displacement response due to an impact force.](image)

Figure 3 shows the surface responses corresponding to various types of impact forces in
which impact durations for all forces are the same, \( \tau_D = 2 \). In figure 3(a), responses due to a half-sine function, a triangle function, and an impulse function obtained from matching the shape of the force function with that of an arbitrarily selected impact created by hitting a concrete surface with a PCB 086C03 hammer. It is clear that the trough-to-ending duration, \( \tau_d \), associated with the half-sine and the triangle functions equal to the impact duration \( \tau_D \) since the arrival and ending time of the impact forces can be easily recognized by the abrupt changes of curves. On the other hand, a more realistic force function, denoted as “hammer” in the figure, leads to a smoother trough in the response curve and thus relatively smaller values for \( \tau_d \). The reason why response curves show abrupt changes at trough and ending points is that the slopes at the beginning and the ending of the force functions are not continuous. In figure 3(c), responses curves due to half-sine-squared and half-sin\(^{1.5}\) are compared with that of half-sine. Because the slopes at the starting and ending points of the sin\(^2\) and sin\(^{1.5}\) functions are continuous, their corresponding response curves appear to have smoother troughs as that shown in the realistic case.

![Graphs](image)

Fig. 3 : Surface responses due to various types of force functions for \( \tau_D = 2 \) and \( \nu = 0.18 \).

The trough-to-ending duration, \( \tau_d \), the compressive duration, \( \tau_0 \), the peak amplitude, \( A_{\text{peak}} \), and
the trough amplitude, $A_t$, associated with various force functions are summarized in table 1. It can be seen from table 1 that the realistic case is close to and falls between the half-sine-squared and half-sin$^{1.5}$ cases. As far as the amplitudes $A_{\text{peak}}$ and $A_t$ are concerned, the half-sine-squared force function seems an appropriate approximation for the realistic impulse that we arbitrarily selected for this work. Therefore, the half-sine-squared function is also selected to illustrate the variation of duration differences with the size of impact duration $\tau_D$, as shown in table 2.

Table 1
Comparisons between responses due to various force functions ($\tau_D = 2, \nu = 0.18$)

<table>
<thead>
<tr>
<th>Force function</th>
<th>$\tau_d$</th>
<th>$\tau_0$</th>
<th>$A_{\text{peak}}$</th>
<th>$A_t$</th>
<th>$A_t / A_{\text{peak}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle</td>
<td>2</td>
<td>1.841</td>
<td>1.159</td>
<td>-0.159</td>
<td>0.137</td>
</tr>
<tr>
<td>Half-sine</td>
<td>2</td>
<td>1.848</td>
<td>1.046</td>
<td>-0.247</td>
<td>0.236</td>
</tr>
<tr>
<td>Half-sin$^{1.5}$</td>
<td>1.939</td>
<td>1.766</td>
<td>1.066</td>
<td>-0.112</td>
<td>0.105</td>
</tr>
<tr>
<td>Half-sine-squared</td>
<td>1.864</td>
<td>1.692</td>
<td>1.084</td>
<td>-0.084</td>
<td>0.078</td>
</tr>
<tr>
<td>Realistic(hammer)</td>
<td>1.918</td>
<td>1.732</td>
<td>1.083</td>
<td>-0.063</td>
<td>0.058</td>
</tr>
</tbody>
</table>

Table 2
Variation of various ratios with impact duration $\tau_D$ (half-sine-squared force, $\nu = 0.18$)

<table>
<thead>
<tr>
<th>Impact duration</th>
<th>$\tau_d / \tau_D$</th>
<th>$\tau_0 / \tau_D$</th>
<th>$\tau_0 / \tau_d$</th>
<th>$A_t / A_{\text{peak}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_D = 10$</td>
<td>0.984</td>
<td>0.965</td>
<td>0.98</td>
<td>0.004</td>
</tr>
<tr>
<td>$\tau_D = 5$</td>
<td>0.961</td>
<td>0.931</td>
<td>0.969</td>
<td>0.015</td>
</tr>
<tr>
<td>$\tau_D = 2$</td>
<td>0.932</td>
<td>0.846</td>
<td>0.908</td>
<td>0.078</td>
</tr>
<tr>
<td>$\tau_D = 1$</td>
<td>0.899</td>
<td>0.764</td>
<td>0.850</td>
<td>0.203</td>
</tr>
<tr>
<td>$\tau_D = 0.5$</td>
<td>0.879</td>
<td>0.703</td>
<td>0.800</td>
<td>0.372</td>
</tr>
<tr>
<td>$\tau_D = 0.2$</td>
<td>0.878</td>
<td>0.673</td>
<td>0.767</td>
<td>0.532</td>
</tr>
<tr>
<td>$\tau_D = 0.1$</td>
<td>0.878</td>
<td>0.673</td>
<td>0.767</td>
<td>0.560</td>
</tr>
</tbody>
</table>

To establish the relations between impact duration $\tau_D$ and response amplitudes $A_{\text{peak}}$ and $A_t$, responses due to half-sine-squared impacts of different durations are plotted in figure 4. From figure 4(a), it can be observed that as impact duration increases, the peak amplitude $A_{\text{peak}}$ approaches $A_0$, while the trough amplitude $A_t$ becomes less significant. For example, the ratios of $A_t$ to $A_{\text{peak}}$ are 0.203, 0.372 and 0.532 for impact duration $\tau_D$ equal to 1, 0.5 and 0.2, respectively. To have a better presentation of the trend of response curves, figure 4(b) shows the variation of amplitudes with horizontal axis in logarithmic scale. The limiting amplitude ratio ($A_t / A_{\text{peak}}$) for very small impact duration is about 0.56 as $\tau_D$ being smaller than 0.14. Table 2 lists the duration ratios and amplitude
ratios for different values of impact duration $\tau_D$. It is interesting to note that the duration ratios appear to be fixed as the force duration becomes smaller, for instance when dimensionless duration $\tau_D$ is as small as 0.2 or less. On the other hand, when dimensionless duration $\tau_D$ is large, the trough amplitude $A_t$ is negligible and impact duration $\tau_D$ can be roughly approximated by $\tau_d$ or $\tau_0$.

![Figure 4: Surface responses due to half-sine-squared forces with $\nu = 0.18$.](image)

Table 3 lists the $(A_t / A_{peak})$ amplitude ratios associated with different values of $\tau_D$ for various force functions. It is apparent that all force functions show similar trends regarding the variation of the $(A_t / A_{peak})$ ratio versus impact durations. In all cases, the ratios associated with $\tau_D = 0.1$ already equal to the limiting values obtained using very small $\tau_D$. The realistic force function recovered from the PCB hammer gives ratios lower than those obtained from the three half-sine-powered functions. Generally speaking, the ratios by the half-sine-squared function agree with the realistic ones for large values of $\tau_D$. For smaller values of $\tau_D$, results by the realistic force appear to be about 70% to 80% of those by the half-sine-squared force.

<table>
<thead>
<tr>
<th>$A_t / A_{peak}$</th>
<th>$\sin$</th>
<th>$\sin^{1.5}$</th>
<th>hammer</th>
<th>$\sin^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_D = 10$</td>
<td>0.050</td>
<td>0.010</td>
<td>0.005</td>
<td>0.004</td>
</tr>
<tr>
<td>$\tau_D = 5$</td>
<td>0.010</td>
<td>0.029</td>
<td>0.015</td>
<td>0.015</td>
</tr>
<tr>
<td>$\tau_D = 2$</td>
<td>0.236</td>
<td>0.105</td>
<td>0.059</td>
<td>0.078</td>
</tr>
<tr>
<td>$\tau_D = 1$</td>
<td>0.416</td>
<td>0.234</td>
<td>0.141</td>
<td>0.203</td>
</tr>
<tr>
<td>$\tau_D = 0.5$</td>
<td>0.615</td>
<td>0.405</td>
<td>0.261</td>
<td>0.372</td>
</tr>
<tr>
<td>$\tau_D = 0.2$</td>
<td>0.803</td>
<td>0.579</td>
<td>0.400</td>
<td>0.532</td>
</tr>
<tr>
<td>$\tau_D = 0.1$</td>
<td>0.864</td>
<td>0.620</td>
<td>0.440</td>
<td>0.560</td>
</tr>
</tbody>
</table>

From figure 4(b), as impact duration $\tau_D$ increases, the peak amplitude $A_{peak}$ shows a
decreasing trend for small values of $\tau_D$ and approaches a constant, $A_0$, for larger values of $\tau_D$. To have a better idea on how peak amplitude varies with impact duration, it is shown in figure 5 the $(A_{\text{peak}} / A_0)$ versus $(\tau_D)$ curves with half-sine force function for various Poisson’s ratios. Since figure 5 is plotted using logarithmic scales, the negative slope of minus one-half within very small values of $\tau_D$ represents that peak amplitude $A_{\text{peak}}$ attenuates with $r^{-0.5}$. On the other hand, the peak amplitude $A_{\text{peak}}$ equals $A_0$ and thus attenuates with $r^{-1}$ for the region of larger $\tau_D$. As shown in figure 5, curves associated with different values of Poisson’s ratios result in almost parallel lines of similar slopes for smaller $\tau_D$ and converge to a single curve for larger $\tau_D$. To investigate the differences among $(A_{\text{peak}} / A_0)$ versus $(\tau_D)$ curves of different force functions, figure 6 shows the corresponding curves of four kinds of force functions for a given Poisson’s ratio $\nu = 0.18$. It is shown in figure 6 that three half-sine related curves exhibit similar trends. In addition, higher-powered sine functions appear to cause larger peak amplitudes. The triangle force function gives the highest values among all for $\tau_D$ larger than 0.5 but leads to results close to those of half-sine $^{1.5}$ for region of very small $\tau_D$.

![Fig. 5: Amplitude ratios for half-sine curve with different Poisson’s ratios.](image)

![Fig. 6: Amplitude ratios for different force functions with Poisson’s ratio $\nu = 0.18$.](image)

### 4. Recovery of impact force functions

To reasonably recover the force function of a specific surface response, one has to identify first the amplitude and the duration of the impact force. In the following, we propose a simple procedure using amplitude curves such as those shown in figure 6 to estimate the force amplitude. Alternatively, the impact force can be simulated as an equivalent force using a relatively simple function, i.e. a half-sine function, or using a portion of the surface signal. The first approach requires an appropriate duration for the simulated force while the latter method utilizes the compressive portion of the surface response as the force function, in which the force duration is $\tau_0$. 


Regarding the determination of the duration of an equivalent half-sine force, the relation between an appropriate value and the duration parameters $\tau_d$ and $\tau_0$ is studied. It is found that such an equivalent half-sine force can reproduce similar frequency contents as those of the original impact functions and can thus provide an alternative way to compute the transfer function associated with a specific impact-echo response. Details on the determinations of force amplitude and equivalent duration of the impact are discussed as follows.

Determination of force amplitude

A realistic impact may be simulated using half-sine functions with power orders between 1.5 and 2, such as the example illustrated in table 1. In all cases, the duration parameter $\tau_0$ is always smaller than $\tau_D$ and the value of $\tau_d$ is ranged between $\tau_0$ and $\tau_D$. Using $\tau_d$ or $\tau_0$ to represent the impact duration would under-estimate the real impact duration, $\tau_D$. As a result, when using a smaller $\tau_D$ to establish the response amplitude $A_{\text{peak}}$ from the half-sine$^{1.5}$ and half-sine-squared curves, higher estimates can be anticipated since $A_{\text{peak}}$ decreases with $\tau_D$ as shown in figure 6. On the other hand, the half-sine curve gives somehow lower values than the two higher order half-sine-powered curves. Without precise computations, using half-sine curves to establish $A_{\text{peak}}$ may roughly balance the over-estimate caused by the designated smaller value for $\tau_D$. The variations of the two response durations with impact duration $\tau_D$ are shown in table 4. The corresponding predictions of amplitude ratios from the “lower” half-sine curve are shown in table 5. To have a better look at the trends of the amplitude predictions, figure 7 shows the predicted amplitude ratios with markers for each impact force. According to results associated with these three cases of impact forces, it is found that the lower ratios associated with $\tau_d$ are very close to the half-sin$^{1.5}$ curve. While the higher ratios with respect to $\tau_0$ seem basically follow the trend of the hammer curve but keep increasing as $\tau$ decreases and finally deviate from the trend. As a result, extracting amplitude ratios from the half-sine curve with durations of $\tau_d$ may provide lower bound values regardless of the types of force functions. On the other hand, the predicted values using $\tau_0$ are higher than the true solutions, especially for smaller impact durations. However, as long as impact duration $\tau_D$ is larger than 1, the amplitude ratios predicted by half-sine agree well with the real values for all three force functions. Owing to the fact that the use of smaller response durations $\tau_d$ and $\tau_0$ would compensate the under-estimated ratios obtained from the half-sine curve, such curve may be used to provide estimates within reasonable ranges for the amplitude ratio ($A_{\text{peak}} / A_0$) associated with various types of force functions.

To simplify the recovery of amplitude ratio ($A_{\text{peak}} / A_0$), the half-sine amplitude curve for $\nu = 0.18$ can be approximated by three simple functions as shown in equation (8) (Cheng et al., 2004).
Variations of $\tau_0$ and $\tau_d$ with $\tau_D$ for half-sin$^2$, half-sin$^{1.5}$ and hammer forces ($\nu = 0.18$)

<table>
<thead>
<tr>
<th>$\tau_D$</th>
<th>$\tau_0/\tau_D$</th>
<th>$\tau_d/\tau_D$</th>
<th>$\tau_0/\tau_D$</th>
<th>$\tau_d/\tau_D$</th>
<th>$\tau_0/\tau_D$</th>
<th>$\tau_d/\tau_D$</th>
<th>$\tau_0/\tau_D$</th>
<th>$\tau_d/\tau_D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_D = 10$</td>
<td>0.965</td>
<td>0.984</td>
<td>0.965</td>
<td>0.990</td>
<td>0.973</td>
<td>0.992</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tau_D = 5$</td>
<td>0.931</td>
<td>0.961</td>
<td>0.940</td>
<td>0.983</td>
<td>0.945</td>
<td>0.983</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tau_D = 2$</td>
<td>0.846</td>
<td>0.932</td>
<td>0.863</td>
<td>0.961</td>
<td>0.867</td>
<td>0.959</td>
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<tr>
<td>$\tau_D = 1$</td>
<td>0.764</td>
<td>0.899</td>
<td>0.807</td>
<td>0.948</td>
<td>0.763</td>
<td>0.926</td>
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<tr>
<td>$\tau_D = 0.5$</td>
<td>0.703</td>
<td>0.880</td>
<td>0.739</td>
<td>0.932</td>
<td>0.666</td>
<td>0.889</td>
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<td></td>
</tr>
<tr>
<td>$\tau_D = 0.2$</td>
<td>0.673</td>
<td>0.878</td>
<td>0.697</td>
<td>0.932</td>
<td>0.614</td>
<td>0.878</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tau_D = 0.1$</td>
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<td>0.878</td>
<td>0.697</td>
<td>0.932</td>
<td>0.614</td>
<td>0.878</td>
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<td></td>
</tr>
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</table>

Table 5
Equivalent values of ($A_{peak}/A_0$) obtained from half-sine curve with $\tau_D$ replaced by $\tau_0$ and $\tau_d$ ($\nu = 0.18$, values associated with $\tau_0$ and $\tau_d$ listed in table 4)

<table>
<thead>
<tr>
<th>$A_{peak}/A_0$</th>
<th>Original values</th>
<th>Estimates by half-sine</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_D$</td>
<td>sin$^2$</td>
<td>sin$^{1.5}$</td>
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<tr>
<td>$\tau_D = 10$</td>
<td>1.004</td>
<td>1.003</td>
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<tr>
<td>$\tau_D = 5$</td>
<td>1.015</td>
<td>1.011</td>
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<tr>
<td>$\tau_D = 2$</td>
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<td>1.066</td>
</tr>
<tr>
<td>$\tau_D = 1$</td>
<td>1.256</td>
<td>1.213</td>
</tr>
<tr>
<td>$\tau_D = 0.5$</td>
<td>1.626</td>
<td>1.549</td>
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<tr>
<td>$\tau_D = 0.2$</td>
<td>2.505</td>
<td>2.392</td>
</tr>
<tr>
<td>$\tau_D = 0.1$</td>
<td>3.539</td>
<td>3.363</td>
</tr>
</tbody>
</table>

Fig. 7: Predictions of amplitude ratios using half-sine curve with $\tau_0$ and $\tau_d$. 
\[ Y = X^{1.4} \quad \text{for } X \leq 0.4 \]
\[ Y = 1 + 0.12X^{-1.7} \quad \text{for } 0.4 \leq X \leq 2.5 \]
\[ Y = 1 \quad \text{for } X \geq 2.5 \]

in which \( X = \frac{\tau_D}{\sqrt{c}} \), \( Y = \frac{A_{\text{peak}}}{A_0} \), and \( c \) is defined in equation (4).

**Determination of impact duration \( \tau_D \)**

As shown in table 4, the ratios of response durations (\( \tau_0 \) and \( \tau_d \)) to impact duration \( \tau_D \) for different force functions are similar, but their differences become more significant as \( \tau_D \) decreases. Figure 8 shows the variations of response durations for different force functions. It is clear that, when the anticipated value of \( \tau_D \) is larger than 2, using response durations \( \tau_d \) and \( \tau_0 \) to predict original impact duration \( \tau_D \) seems possible since differences between curves are minor. However, for cases of small \( \tau_D \), reliable predictions become relatively difficult since precise relations between the response durations and \( \tau_D \) cannot be determined without prior knowledge of the force function.

![Figure 8: Variations of response durations \( \tau_0 \) and \( \tau_d \) with impact duration \( \tau_D \).](image)

**Determination of frequency contents of impact force**

In the previous two sections, we illustrate that the predictions for the amplitude and duration of an impact force can be carried out through their relations with the response durations \( \tau_d \) and \( \tau_0 \). Nevertheless, it is quite clear that one can not possibly recover the exact time history of force function from the established figures and tables. When dealing with frequency spectra, the most significant characteristics of the force function is its frequency contents. Thus, two simple approaches will be introduced to reproduce reasonable frequency contents for arbitrary force functions.

From the time history of a recorded surface response, the most straight-forward way to obtain a force function similar to the real one is to use the compressive response with duration of \( \tau_0 \) as a
“Pseudo” force function. As an example, the comparison is made between frequency spectrum of the original half-sine force function and that of the pseudo force for the case of $\tau_D$ equal 2, as shown in figure 9. For this specific case of $\tau_D$, the pseudo force spectrum agrees well with the real force spectrum since $\tau_0$ is close to $\tau_D$. As $\tau_0$ becomes much smaller than $\tau_D$, it can be expected that the difference between the two spectra would be more significant. In addition, the first zero of the frequency spectrum for the half-sine force is at frequency $1.5/\tau_D$.

![Frequency spectra](image1)

(a) frequency spectra

![Force-time function](image2)

(b) force-time function

Fig. 9: Frequency spectra associated with half-sine force for $\tau_D = 2$.

Similar behavior can be observed from responses due to other force functions, as shown in figure 10. When comparing between the real spectra (dashed curves) shown in figures 9 and 10, it is clear that the ordinate of the half-sine curve at zero frequency is the highest and the frequency of the first near-zero amplitude for the other curves extend to frequencies higher than $1.5/\tau_D$. Owing to their smaller durations, the pseudo curves (solid curves) are always smoother than the real curves (dashed curves), in which the former have lower ordinates at zero frequency and also higher frequencies for the first zero or near-zero amplitude. Such effects may be compensated one way or another by changing the force function to a half-sine function since the half-sine function results in relatively steepening effect among the various force functions. This means that, when replacing the real forces with the half-sine force and a duration smaller than $\tau_D$, the equivalent spectrum may be similar to the real one. In figure 10, the marked curves represent the equivalent spectra computed by a half-sine function with duration $\tau_0$. Except for the triangle force, the equivalent spectra appear better than or at least as good as the pseudo ones in matching the real spectra. The trough-to-ending duration $\tau_d$ is always larger than the compressive duration $\tau_0$ and thus the equivalent spectrum using duration of $\tau_d$ would be steeper than the $\tau_0$ one. In several cases, the equivalent half-sine $\tau_0$ spectra agree well with the real ones in figure 10. Since, for cases where $\tau_0$ almost equals to $\tau_D$, the overestimate by the half-sine force cannot be balanced. The equivalent half-sine $\tau_0$ spectra become much steeper than the real spectra for cases of very large values of $\tau_D$. As a result, the use of an
The equivalent half-sine spectrum with a duration of $\tau_0$ can potentially provide good approximations for the real spectrum, providing the value of $\tau_D$ is relatively large.

The spectra associated with a small impact duration is shown in figure 11, in which the half-sine-squared force has an impact duration $\tau_D$ equals 0.2. Since the differences among $\tau_d$, $\tau_0$ and $\tau_D$ are more significant for smaller values of $\tau_D$, as mentioned in the pervious section. It can be seen that both the pseudo spectrum computed directly from the compressive response of duration $\tau_0$ and the equivalent half-sine spectrum with $\tau_0$ become much less steeper than the real spectrum. The

Fig. 10 : Frequency spectra associated with various force functions for $\tau_D = 2$. 

(a) frequency spectra 
(b) force-time function
equivalent half-sine spectrum obtained with duration $\tau_d$ appears to be steeper than the real spectra. As can be expected, both equivalent spectra are steeper than the pseudo spectrum. A smoother curve has smaller amplitude within the lower frequency region and larger amplitude within the higher frequency region. When the frequency associated with the point of intersection between a smoother curve and a steepled curve is defined as “trend-reverse” frequency, the trend-reverse frequency seems to be about half of the frequency of the first near-zero amplitude, in the range between $0.75/\tau_D$ and $1/\tau_D$. Apparently, the equivalent half-sine spectra with $\tau_d$ and $\tau_0$ may serve as the upper and lower bounds of the real spectrum. Another example is shown in figure 12, where the equivalent half-sine spectra for the half-sin$^{1.5}$ force is plotted for $\tau_D = 0.2$. The $\tau_0$ curve approaches closely to the lowered pseudo spectrum while the $\tau_d$ curve appears to be still higher than the real spectrum. As $\tau_D$ decreases the half-sine $\tau_0$ curve moves towards the pseudo spectrum, and the half-sine $\tau_d$ curve is steeper than but approaches downward to the real spectrum.

![Graphs showing frequency spectra and force-time function](image)

Fig. 11 : Frequency spectra associated with half-sine-squared force for $\tau_D = 0.2$.

Based on the above findings, we can conclude that, when $\tau_D$ is large, the real spectrum would fall between the equivalent half-sine spectrum of $\tau_0$ and the pseudo spectrum defined by the compressive duration $\tau_0$. The former could be steeper while the latter is slightly smoother than the real spectrum. On the other hand, for very small $\tau_D$, the equivalent spectrum of $\tau_d$ seems to be the closest one of all to the real spectrum. For moderate values of $\tau_D$, the real spectrum obviously
locates somewhere between the two equivalent half-sine spectra. To illustrate this point, we arbitrarily assume that the duration $\tau$ used in computing the equivalent half-sine spectrum linearly varies with the amplitude ratio ($A_t/A_{\text{peak}}$). The amplitude ratio is within zero and the maximum ratio listed in the bottom row of table 3. The equivalent half-sine spectrum with a duration $\tau$ linearly interpolated between $\tau_0$ and $\tau_d$ is shown in figure 12(d). As can be expected, this equivalent spectrum is somewhat better than the half-sine $\tau_d$ curve in matching the real spectrum.

![Figure 12: Frequency spectra associated with half-sin$^{1.5}$ force for $\tau_D = 0.2$.](image)

The second example of such interpolated spectrum where a moderate $\tau_D$ of 1 is selected is shown in figure 13. From table 3, we recognize that ($A_t/A_{\text{peak}}$) ratio is 0.234 and the extreme ratios associated with $\tau = \tau_0$ and $\tau = \tau_d$ are 0 and 0.62, respectively. From table 5, the corresponding values for $\tau_0/\tau_D$ and $\tau_d/\tau_D$ are 0.807 and 0.948. The equivalent duration can be obtained from interpolating 0.234 between 0 and 0.62. Thus the resulting duration equals to $\tau = 0.86\tau_D$. Again, it is obvious that the interpolated spectrum agrees well with the real spectrum.

For practical Impact-Echo tests on concrete surface, the dimensionless impact duration $\tau$ for the near-source receiver (typically a radial distance of 3-cm) is normally higher than 2 and seldom less than 1. Thus, the only parameter we need to normalize an Impact-Echo spectrum is simply the compressive duration $\tau_0$. Neither $\tau_d$ nor the interpolation process is required in recovering a simulated transfer function associated with an impact test.
Computation of simulated transfer function

Consider two normalized functions $f_u(t)$ and $W_u(t)$ with their maximum amplitudes equal to a unit amplitude. The force and response functions are expressed as $f(t) = Z f_u(t)$ and $W(t) = Z A_{\text{peak}} W_u(t)$, respectively. Since the amplitude of the equivalent force function is equal to that of $W(t)$, that is $Z A_{\text{peak}}$ or $ZA_0$, the equivalent force function can be expressed as $f_e(t) = Z A_{\text{peak}} f_{ue}(t)$. $f_{ue}(t)$ is a pseudo force function, such as the half-sine function or the compressive response corresponding to $W_u(t)$, of unit amplitude. The transfer function can then be expressed in terms of the Fourier transform pairs of $f_u(t)$ and $W_u(t)$ as

$$\frac{\hat{W}(f)}{\hat{f}(f)} = \frac{Z A_{\text{peak}} \hat{W}_u(f)}{Z \hat{f}_u(f)} = Z A_{\text{peak}} \frac{\hat{W}_u(f)}{\hat{f}_{ue}(f)} = Y A_0 \frac{\hat{W}(f)}{\hat{f}_e(f)}$$

Equation (9) indicates that the ordinate of transfer function is of the order of $A_0$ and thus depends on radial distance $r$, shear modulus $G$, and Poisson’s ratio $\nu$. Test data of approximately the same mass density and Poisson’s ratio, as in the case for concrete specimens, can be compared based on the following conditions. Assume that $r = 0.03 \text{m}$ and $C_p = 4000 \text{m/s}$. A relatively long impact duration
is also assumed such that $Y = 1$. The default transfer function can then be expressed as

$$\frac{W(f)}{f_n(f)} = A_0(r, C_p)\text{, default} \quad \frac{\tilde{W}_u(f)}{\tilde{f}_u(f)} = A_0(0.03, 4000) \quad \frac{\tilde{W}_u(f)}{\tilde{f}_u(f)}$$

with an ordinate constant $A_0(0.03, 4000)$. $A_0$ is inversely proportional to $r$ and the square of $C_s$ (also $C_p$) as shown in equations (2) and (3). A scaled transfer function can be obtained as

$$\frac{W(f)}{f_n(f)} = A_0(r, C_p)\text{, default} = Y \left( \frac{A_0(r, C_p)}{A_0(0.03, 4000)} \right) \quad \frac{\tilde{W}_u(f)}{\tilde{f}_u(f)} = F_n \left( \frac{\tilde{W}_u(f)}{\tilde{f}_u(f)} \right) \approx F_n \left( \frac{\tilde{W}_u(f)}{\tilde{f}_u(f)} \right)$$

where $F_n = Y \left( \frac{4000}{C_p} \right)^2 \times \left( \frac{0.03}{r} \right)$

Accordingly, we define a simulated transfer function which is computed from the surface response function $W(t)$, the equivalent force function $f_e(t)$, and a scale constant $F_n$.

The frequency counterpart of the equivalent force function $f_e(t)$ can be obtained via computer programs using FFT algorithm. However, the amplitude of the Fourier transform for a unit half-sine function with duration $t_D$ can be explicitly expressed as,

$$|F(f)| = 2 \cos(\pi t_D f) \frac{t_D}{\pi(1-4t_D^2 f^2)}$$

with $f$ representing the sampling frequency. The FFT computation can thus be omitted when deducing the equivalent half-sine spectra.

### 5. Experimental Application

In this section, the application of the proposed approach to the quantification of Impact-Echo signals is illustrated with experimental data. Two examples are presented in which the first one contains impact tests at an identical location using three different impact sources, while the second case is associated with impact tests at different points on a uniform plate. It should be noted that the differences among thickness amplitudes resulted from repeated impacts for a specific steel ball are typically within 5% and could be as high as 10% for the smallest ball, the 3-mm ball. The test data shown in the first example are intentionally selected so that the thickness amplitudes recovered using pseudo forces for the three different steel balls can be almost identical. In the second example, six impact tests were performed at each location with a specific steel ball. The curves shown in figures are those corresponding to the test sets which give the intermediate value among the six impacts.
Example 1:

In this example, a 60-cm×60-cm concrete plate specimen with 15-cm thickness is prepared and repeatedly tested at a pair of fixed points with steel balls of different diameters. The distance between the impact point and the near-source receiver is 3 cm. The P-wave velocity calculated from the first arrivals of the surface response histories at two receivers is 3970 m/s. The diameters of balls are 3, 5, and 6 mm and their corresponding compressive durations are 16, 29, and 36 µs, which are equivalent to $\tau_0$ of about 1.38, 2.52, and 3.10, respectively. The recorded time histories of the impact responses and their frequency amplitude spectra are shown in figure 14. It is apparent that both time and frequency responses associated with three different balls are very similar in terms of the major waveform and of the frequency contents near the thickness frequency. Since the frequency amplitude is a direct function of force amplitude and duration, the ordinates at the thickness frequency for different impacts are clearly different from each other.

To illustrate the effect of using the half-sine amplitude curve in figure 6 to generate simulated transfer functions, the three amplitude spectra in figure 14 are normalized with respect to the pseudo impulses as shown in figure 15. The scaled thickness amplitudes associated with all three impact sources appear to be very consistent. Although, what we illustrate in the figures are selected to have very close scaled thickness amplitudes, these promising experimental data can be easily achieved and also be expected since all three curves are obtained by impacting at an identical point. The values shown in parentheses, on the other hand, represent the scaled thickness amplitudes which are calculated using half-sine impulses with duration $\tau_0$. The differences among values in the parentheses are less than 4%. The trend-reverse frequencies associated with the three impact tests can be estimated by $0.75/\tau_0$ and are about 47, 26, and 21 kHz, respectively. Since the thickness frequency, 12.45 kHz, is much lower than the trend-reverse frequencies, the thickness amplitudes recovered by the half-sine force are clearly lower than those obtained with pseudo forces as can be expected. The recovered thickness amplitudes basically remain constant for different sizes of steel balls regardless of the (pseudo or equivalent half-sine) forces used. Figure 16 shows the force-time functions associated with the pseudo forces and the equivalent half-sine forces. It is obvious that, as impact duration becomes larger, the skewness of the curves becomes more significant.
(a) 3-mm steel ball and 16-µs duration ($\tau_0 = 1.38$).

(b) 5-mm steel ball and 29-µs duration ($\tau_0 = 2.52$).

(c) 6-mm steel ball and 36-µs duration ($\tau_0 = 3.1$).

Fig. 14: Response wave forms and amplitude spectra with respect to three different impact steel balls (impacting at an identical point).
Fig. 15: Simulated transfer functions with respect to three different impact steel balls.

(a) 3-mm steel ball and 16-µs duration.  
(b) 5-mm steel ball and 29-µs duration.  
(c) 6-mm steel ball and 36-µs duration.

Fig. 16: Simulated force-time functions with respect to three different impact steel balls.

(a) 3-mm steel ball and 16-µs duration.  
(b) 5-mm steel ball and 29-µs duration.  
(c) 6-mm steel ball and 36-µs duration.
Example 2:

In the second example, impact tests are performed on an 80-cm×80-cm concrete plate specimen with a thinner thickness, 9-cm. Three sizes of balls, 3-mm, 5-mm and 6-mm in diameters, are used as impact sources at three different points, specified as A, B, and C. The distance between the impact point and the receiver is kept 3 cm. The P-wave velocity recovered from the first arrivals of the surface response histories at two receivers is about 3750 m/s. The corresponding compressive duration $t_0$ are about 15, 36 and 41 $\mu$s ($\tau_0 = 1.19, 2.93, 3.34$), respectively. The estimated trend-reverse frequencies $(0.75/\tau_0)$ of the force spectra are around 51, 21 and 18 kHz.

It is shown in figure 17 the time histories and frequency amplitudes of the surface displacement responses for three specific test sets. Comparing to what we saw in the previous example, the differences in both wave forms and frequency amplitudes of the three impact tests are more obvious since they are recorded at different locations. The thickness frequency amplitude for the 5-mm steel ball are about 85% and 50% higher than those of the 3-mm and 6-mm steel balls. After scaling the frequency spectra with the proposed pseudo force spectra, the simulated transfer functions of the three impact tests provide consistent values for amplitudes at the thickness frequency. The differences among thickness amplitudes are less than 5%, as shown in figure 18. The values in parentheses stand for the frequency amplitudes obtained using the half-sine approach. Note that the amplitude of the 3-mm (15$\mu$s) is smaller than those of the 5-mm (36$\mu$s) and the 6-mm (41$\mu$s). The near identical amplitudes in figures 18(b) and 18(c) are due to the fact that corresponding force spectra have almost the same ordinate near the trend-reverse frequency.

What worth of mentioning is that the thickness amplitude caused by the real force might be very close to the constant value shown in figure 18(b) or 18(c) since the force-frequency amplitude of the real force could be close to the estimated ones near the trend-reverse frequency. The differences among these three values obtained from the half-sine approach are around 7% which is slightly larger than that of the pseudo approach, 4%. Moreover, the thickness frequencies 20.5 kHz and 19.77 kHz indicate that the recovered thicknesses of the plate, 8.8 cm and 9.1 cm, are close to the designated 9-cm. Consequently, the two proposed procedures can both provide reasonably consistent thickness amplitudes in the simulated transfer function, even when surface responses are generated using different impact sources at different locations on the plate surface.
(a) impact responses at point A using 3-mm steel ball (15-μs duration).

(b) impact responses at point B using 5-mm steel ball (36-μs duration).

(c) impact responses at point C using 6-mm steel ball (41-μs duration).

Fig. 17: Surface displacement responses by impacting three steel balls at different locations.
6. Conclusions

The analytical solution for the normal surface displacement due to the Heaviside force has been reviewed and applied to compute the surface responses caused by various impulse-type force functions. A series of numerical studies on the direct Rayleigh response are carried out to analyze the variations of the response amplitudes and the response durations associated with the direct wave form. Based on the varying trends constructed by half-sine-powered forces, it can be concluded that the simple half-sine function with duration $\tau_0$ provides frequency contents similar to those of the designated force functions. Accordingly, an equivalent force function can be easily established with amplitude curves that are numerically obtained using half-sine function as the impact force. In order to make comparison between different impact responses, a simulated transfer function is proposed, as equation (11). Amplitudes at the thickness frequency can now be properly scaled and also remain constant for a uniform specimen.

In computing the normal surface displacement, numerical results are obtained by
superposing responses calculated based on the analytical solution associated with the Heaviside step force. The well-known Pekeris’ explicit solution for Poisson’s ratio of 0.25 is extended to a more general expression which is valid for Poisson’s ratio of arbitrary values. Such explicit formulation may potentially be useful to the investigation on related research topics.

For the construction of relationship between response amplitude and force duration, various force functions all lead to similar trends. These trends can be separated into three regions such that approximations can be made by three simple mathematic expressions, as shown in equation (8). For a given radial distance $r$, relatively small durations result in attenuation of amplitude of $r^{-0.5}$, while relatively long durations cause amplitudes attenuated with $r^{-1}$.

Because the value of a transfer function is dependent on radial distance, shear modulus, and Poisson’s ratio, a scaled transfer function is proposed as shown in equation (11). The scale factor in equation (11) is the ordinate constant of the transfer function associated with a designated test situation where radial distance is 0.03 m, P-wave velocity is 4000 m/s, and impact duration is relatively large. Such factor is equivalent to the nominal amplitude factor $A_0$ defined in equation (3) and can be arbitrarily adjusted according to the attribute of tests. The advantage of using a scaled transfer function is that the ordinate of thickness amplitude can be of the order near unity. Thus, it provides more manifest intuition than that of the real transfer function.

The realistic impact is usually highly field-sensitive and the compressive portion of surface response essentially retains the shape of the impact force. Using a pseudo force function is comparatively better than using the equivalent half-sine force in providing stable values for the thickness amplitudes. However, the half-sine approach allows one to deduce transfer functions without the need of FFT computation. Thus, the half-sine approach is a relatively simpler alternative to calculate the normalized amplitude at the thickness frequency.

From the two application examples of Impact-Echo experiments, it is found that the proposed approach can recover steady values for the thickness amplitude not only for tests at a fixed location but also for tests using various impact sources at different locations. When thickness amplitudes obtained for an identical specimen remain constant regardless of the impact sources and locations, the thickness amplitude can serve as an index to evaluate the impedance change at the bottom interface of the specimen. The simulated transfer functions computed for two groups of experimental data demonstrate the capability of the proposed method in quantifying the thickness amplitude for Impact-Echo tests. The results show that the proposed approach is very promising and
can be extended to evaluate the information of substrate layers.

References


Appendix   List of Symbols

\( A_0 \) : permanent normal surface displacement due to Heaviside step force
\( A_{\text{peak}} \) : compressive peak amplitude of the direct Rayleigh waveform
\( A_t \) : trough amplitude of the direct Rayleigh waveform
\( a, b, c \) : roots of Rayleigh characteristic equation \( R(y^2) = 0 \), \( c \) is the largest root
\( C_p, C_s, C_R \) : P, S, and Rayleigh wave velocities
\( F(f) \) : Fourier transform of a unit half-sine function
\( F_n \) : scale constant of simulated transfer function
\( f \) : sampling frequency
\( f(t) \) : force-time function
\( f_a(t) \) : normalized function of \( f(t) \) with a unit amplitude
\( f_{\text{ue}}(t) \) : approximation function of \( f_a(t) \)
\( G \) : shear modulus
\( \rho \) : mass density
\( \gamma \) : ratio of S-wave velocity to Rayleigh wave velocity (\( = \sqrt{c} \))
\( H(t) \) : Heaviside step function
\( \mu \) : Lame’s constant = \( G \)
\( \nu \) : Poisson’s ratio
\( R(y) \) : characteristic equation of Rayleigh wave
\( r \) : radial distance
$\sigma$: ratio of S-wave velocity to P-wave velocity

t: elapsed time

$t_0$: duration of compressive portion of the direct Rayleigh wave form

$t_D$: impact duration

$t_d$: trough-to-ending duration of the direct Rayleigh wave form

$\tau$: dimensionless time parameter ($=Cst/r$)

$\tau_0$: dimensionless compressive response duration ($=Cst_0/r$)

$\tau_D$: dimensionless impact duration ($=Cst_D/r$)

$\tau_d$: dimensionless trough-to-ending response duration ($=Cst_d/r$)

$W(t)$: normal surface response due to force-time function

$W_u(t)$: normalized function of $W(t)$ with a unit amplitude

$w(t)$: normal surface response due to the Heaviside step function

$X$: ratio of impact duration to the R-wave arrival time ($=\tau_D/\sqrt{c}$)

$x, \tilde{x}$: Fourier transform pair

$Y$: ratio of $A_{\text{peak}}$ to $A_0$

$y$: dimensionless variable defined as $C_s/C_R$

$Z$: amplitude of the Heaviside step force
List of Figures

Fig. 1 : Vertical displacement at free surface according to equation (3).

Fig. 2 : A typical surface displacement response due to an impact force.

Fig. 3 : Surface responses due to various types of force functions for $\tau_D = 2$ and $\nu = 0.18$.

Fig. 4 : Surface responses due to half-sine-squared forces with $\nu = 0.18$.

Fig. 5 : Amplitude ratios for half-sine curve with different Poisson’s ratios.

Fig. 6 : Amplitude ratios for different force functions with Poisson’s ratio $\nu = 0.18$.

Fig. 7 : Predictions of amplitude ratios using half-sine curve with $\tau_0$ and $\tau_d$.

Fig. 8 : Variations of response durations $\tau_0$ and $\tau_d$ with impact duration $\tau_D$.

Fig. 9 : Frequency spectra associated with half-sine force for $\tau_D = 2$.

Fig. 10 : Frequency spectra associated with various force functions for $\tau_D = 2$.

Fig. 11 : Frequency spectra associated with half-sine-squared force for $\tau_D = 0.2$.

Fig. 12 : Frequency spectra associated with half-sin$^{1.5}$ force for $\tau_D = 0.2$.

Fig. 13 : Frequency spectra associated with half-sin$^{1.5}$ force for $\tau_D = 1$.

Fig. 14 : Response wave forms and amplitude spectra with respect to three different impact steel balls (impacting at an identical point).

Fig. 15 : Simulated transfer functions with respect to three different impact steel balls.

Fig. 16 : Simulated force-time functions with respect to three different impact steel balls.

Fig. 17 : Surface displacement responses by impacting three steel balls at different locations.

Fig. 18 : Simulated transfer functions of the three impacts corresponding to figure 17.
List of Tables

Table 1: Comparisons between responses due to various force functions ($\tau_D = 2$, $\nu = 0.18$)
Table 2: Variation of various ratios with impact duration $\tau_D$ (half-sine-squared force, $\nu = 0.18$)
Table 3: Variation of amplitude ratios ($A_t / A_{peak}$) with impact duration $\tau_D$ ($\nu = 0.18$)
Table 4: Variations of $\tau_0$ and $\tau_d$ with $\tau_D$ for half-sin$^2$, half-sine$^{1.5}$ and hammer forces ($\nu = 0.18$)
Table 5: Equivalent values of ($A_{peak} / A_0$) obtained from half-sine curve with $\tau_D$ replaced by $\tau_0$ and $\tau_d$ ($\nu = 0.18$, values associated with $\tau_0$ and $\tau_d$ listed in table 4)