Anticipatory Anxiety and the Annuity Puzzle

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Abstract

The “annuity puzzle,” which describes the inconsistency between the theoretical prediction of full annuitization and the low rate of annuitization in reality has recently been widely discussed in the literature. In this paper, we propose that anticipatory anxiety is one of the rationales for the annuity puzzle. By setting up a two-period model under anticipatory anxiety, which is a decreasing function of the average of future consumption and an increasing function of the riskiness of future consumption as modeled by Caplin and Leahy (2001), we find that the optimal conditions for partial annuitization as well as the optimal annuitization level are sensitive to the degree of anxiety. If purchasing annuities can reduce the disutility from anxiety, then the individual will decrease the investment in the annuity when his/her degree of anxiety increases. Moreover, given reasonable parameters, our simulation results show that anxiety can explain why individuals annuitize their income at a relatively low level.

JEL classification: D14, G11, G22, J26

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I. Introduction

Aging has become a critical issue in many developing countries. People who live longer than they expected to but who do not save enough for their old age could become a tremendous burden for the society. Thus, the individual’s saving and hedging behavior in relation to longevity risk have received much attention both in industrial circles and in academia.

When the only uncertainty that an individual faces is the time of death, Yaari (1965) predicted that without a bequest motive, the individual that maximizes expected utility should fully annuitize his/her income when annuity products are priced as being actuarially fair. After relaxing the assumptions in Yaari (1965), Davidoff, Brown, and Diamond (2005) found that the result of full annuitization still holds for a large set of individual preferences. Even when there is serious mismatch between the consumption trajectory and the annuity income path, it is still optimal to annuitize at least two-thirds of an individual’s assets. However, very few people in the real world fully annuitize their income. In fact, studies show that most people annuitize less than 30% of their income (Blake, 1999; Finkelstein and Poterba, 2002). The literature has referred to the inconsistency between the theoretical prediction regarding the purchase of an annuity and the low rate of annuitization in reality as the “annuity puzzle”. The purpose of this paper is to provide a rationale for the annuity puzzle.

Several papers in the literature have provided rationales for the annuity puzzle from different perspectives. In addition to bequest motives (Yaari, 1965; Bernheim, 1991), the explanations for limited annuitization include adverse selection (Friedman and Warshawsky, 1988), the crowding out effect due to the existence of Social Security or other employer-sponsored pension plans (Mitchell et al., 1999; Brown and Poterba, 2000) and the high administrative loading factors of annuity products
(Mitchell el al., 1999). The ability of families to pool risk and thus substitute for a private annuity market can also downsize the annuity market. (Brown and Poterba, 2000). Babbel and Merrill (2006) show that individuals annuitize a small part of their income because they are worried about the bankruptcy of insurance companies. Although the above papers have provided many insightful rationales, their analyses do not consider the individuals’ anticipatory emotions. Recently, Brown (2007) has demonstrated that the individuals’ behavioral perspectives, such as mental accounting, loss aversion, regret aversion or the illusion of control, play significant roles in relation to annuitization. Consistent with Brown’s (2007) concern about behavioral perspectives, we propose that anticipatory anxiety could be another rationale for the annuity puzzle.

Anticipatory anxiety has been recognized as one of the essential factors influencing the individual’s behavior in the sociology and psychology literature (e.g., Loewenstein, 1987). Recently, researchers have further demonstrated that anticipatory anxiety could affect the individual’s economic and financial decisions. For example, Caplin and Leahy (2001) show that the anticipatory anxiety could explain both the equity premium puzzle and the risk free rate puzzle. Caplin and Leahy (2004) and Epstein (2008) argue that anticipatory anxiety will affect the individual’s demand for information. Huang (2008) shows that an individual who is characterized by anticipatory anxiety may demand less or more insurance than the traditional expected utility maximizer. In this paper, we intend to link the relationship between anticipatory anxiety and the annuity puzzle. When the individual’s preference is characterized by anticipatory anxiety, we will show that the individual may not fully annuitize and, more importantly, may invest a small proportion of his/her wealth in annuity products.

In this paper, we adopt a two-period model as in Davidoff, Brown, and Diamond (2005). We assume that the representative individual will definitely be alive in the
first period and will face mortality risk in the second period. To transfer his/her income between two periods, s/he can purchase an annuity with a certain payoff or invest in a bond in the first period. For simplicity, we assume that the utility function is additively separable with the consumption in the first and in the second period. Unlike Davidoff, Brown, and Diamond (2005) who assume a state-independent utility function, we assume that the individual’s utility function is not the same at the state of being alive and at the state of being dead. The utility function at the state of being dead denotes the bequest motive.

To incorporate anticipatory anxiety into the individual’s preferences, we adopt Caplin and Leahy’s (2001) setting in which the anticipatory anxiety is negatively correlated with the average consumption in the second period and positively correlated with the riskiness of the consumption also in the second period. In order to treat the traditional expected utility model as a special case, we further assume that the individual’s utility is additively separable with both consumption and anticipatory anxiety.

From our theoretical model, we find the necessary and sufficient conditions for partial annuitization. The conditions show that, even without a bequest motive, partial annuitization may be optimal when individuals anticipate anxiety. Furthermore, when the degree of anxiety increases, individuals might decrease the investment in annuities if the decrease in the investment will reduce the disutility from anxiety. Investing in the annuity has two effects. On the one hand, it will increase the average consumption level and further decrease the disutility from anxiety. On the other hand, it will also increase the riskiness of future consumption and hence increase the disutility from anxiety. Therefore, whether or not individuals will demand more in terms of an annuity will depend on the relative intensity of the above two effects. If an increase in the annuity will increase the disutility from anxiety, then an increase in the degree of
anxiety will cause the investment in the annuity to decrease. Our theoretical model demonstrates that anticipatory anxiety could explain why people do not fully annuitize their income.

We further use simulation to answer the question as to why people annuitize their income at a low level. In this simulation, the parameters such as the interest rate, the risk premium, the degree of risk aversion and the mortality rate are set according to the setting in the literature (e.g., Babbel and Merrill, 2007). We further assume that the utility function exhibits constant relative risk aversion and that the anxiety function is of an exponential form. We check the optimal annuitization level in the cases both with and without the presence of risky assets. We find that the optimal annuitization ratio is sensitive to the level of anxiety. Specifically, the individuals with relative risk aversion of 3 will annuitize less than 30% of their income when the degree of anxiety equals 0.2.

The remainder of this paper is organized as follows. The anxiety-averse consumption model is constructed in Section 2. Section 3 provides our simulation results. Section 4 concludes the paper.

II. Model Approach

Assume that there are many identical individuals who make asset allocation decisions that involve private annuities based on a perfectly competitive private insurance market and a risk-free bond. For simplicity, we assume that individuals can only live for two periods. In the first period, the representative individual is alive for sure with an initial wealth $w$. In the second period, the individual faces a probability of death $q \in (0,1)$ and all his/her wealth comes from the performance of the investment in the first period.

The individual can allocate part of his/her initial wealth among private annuities,
and risk-free bonds with dollar amounts $A$ and $B$, respectively, in the first period.

The short sell constraint is imposed:

$$A \geq 0 \quad \text{and} \quad B \geq 0$$  \hspace{1cm} (1)

The assets will generate gross returns $R_A$ and $R_B$ in the second period, respectively. As assumed in Davidoff, Brown and Diamond (2005), we assume that the return on the annuity is greater than that on the bonds. Thus,

$$R_A > R_B.$$

If the agent is alive in the second period, then he/she can obtain $R_A A + R_B B$.

Otherwise, the agent will obtain $R_B B$.

Let $U(C_1, C_2, X)$ denote the individual’s lifetime utility function which depends on the consumption level in period $t$ ($C_t$) where $t = 1, 2$, and the anticipatory anxiety is denoted by $X$. By adopting Caplin and Leahy’s (2001) setting, we model anxiety as a function of the average consumption in the second period $\mu$ and the variance of the second period consumption $\sigma^2$. Note that

$$\mu = (1 - q) R_A A + R_B B,$$

and

$$\sigma^2 = q (1 - q) R_A^2 A^2.$$  

Specifically, the lifetime utility function is as follows:

$$U(C_1, C_2, X) = u(w - A - B) + (1 - q) \delta u(R_A A + R_B B) + q \delta v(R_B B) - \kappa X (-\alpha \mu + \beta \sigma^2),$$  \hspace{1cm} (2)

where $u$ denotes the traditional utility function with $u' > 0$ and $u'' < 0$ and $v$ denotes the bequest utility function with $v' > 0$ and $v'' < 0$. As in Davidoff, Brown and Diamond (2005), we assume that zero consumption is extremely bad, i.e.,

$$\lim_{C_t \to 0} u'(C_t) = \lim_{C_t \to 0} v'(C_t) = \infty, \quad t = 1, 2.$$  \hspace{1cm} (3)

In Equation (2), $\delta$ is the discount factor. $\kappa$ is a positive constant as an “anxiety
coefficient”, which is a linear weight on the anxiety function $X$ with $X' > 0$, $X^* \geq 0$ and $X(0) = 0$. $\alpha$ and $\beta$ are positive constants which describe the linear weights of the mean and variance of consumption, respectively, in the anxiety function. Thus, the anxiety function is decreasing in the average consumption and increasing in the riskiness of consumption.

Under the short selling constraint (1), the optimization problem that the anxiety-averse individual is facing is:

$$\max_{A,B} U(C_1, C_2, X) = u(w - A - B) + (1 - q)\delta u(R_A A + R_B B) + q\delta v(R_B B) - \kappa X(-\alpha \mu + \beta \sigma^2).$$

The first derivatives of the optimization problem with respect to $A$ and $B$ are

$$L = \frac{dU}{dA} = -u'(w - A - B) + (1 - q)\delta u'(R_A A + R_B B)R_A$$
$$+ \kappa X'(z)(1 - q)R_A(\alpha - 2\beta qR_A A)$$

and

$$M = \frac{dU}{dB} = -u'(w - A - B) + (1 - q)\delta u'(R_A A + R_B B)R_B + q\delta v'(R_B B)R_B$$
$$+ \kappa X'(z)\alpha R_B,$$

where $z = -\alpha((1 - q)R_A A + R_B B) + \beta q(1 - q)R_A^2 A^2$.

If the second-order condition holds, i.e.,

$$\begin{vmatrix}
\frac{dL}{dA} & \frac{dL}{dB} \\
\frac{dM}{dA} & \frac{dM}{dB}
\end{vmatrix} > 0,$$

where

$$\frac{dL}{dA} = u^*(w - A - B) + (1 - q)\delta u^*(R_A A + R_B B)R_A^2$$
$$- \kappa X^*(z)[(1 - q)R_A(\alpha - 2\beta qR_A A)]^2$$
$$+ \kappa X'(z)[-2\beta q(1 - q)R_A^2]$$

7
\[
\frac{dL}{dB} = \frac{dM}{dA} = u''(w - A - B) + (1 - q)\delta u''(R_A A + R_B B)R_A R_B \\
- \kappa X''(z)\alpha (1 - q)R_A R_B (\alpha - 2\beta q R_A A)
\]

(9)

\[
\frac{dM}{dB} = u''(w - A - B) + (1 - q)\delta u''(R_A A + R_B B)R_B^2 + q\delta v''(R_B B)R_B^2 \\
- \kappa X''(z)(\alpha R_B)^2,
\]

(10)

then the first-order condition is the necessary and sufficient condition for the internal solutions \(A^*\) and \(B^*\). The following Lemma demonstrates the sufficient conditions for the second-order condition:

**Lemma 1** If

1. \(X'' = 0\), or
2. \(X'' > 0\), and, for all \(A\) and \(B\),

\[
0 > \alpha - 2\beta q R_A A > \frac{2[u''(w - A - B) + (1 - q)\delta u''(R_A A + R_B B)R_A R_B]}{\kappa X''(z)\alpha (1 - q)R_A R_B},
\]

(11)

then the second-order condition holds.

**Proof** Please see Appendix A.

The following Proposition shows the conditions for the internal solutions to the optimization problem (4):

**Proposition 1** Suppose the conditions in Lemma 1 hold. The optimization problem (4) has internal solutions \(A^*\) and \(B^*\) if and only if \(L = 0\) and \(M = 0\). Or, equivalently,

\[
u'(w - A^* - B^*) = (1 - q)\delta u'(R_A A^* + R_B B^*)R_A \\
+ \kappa X'(z^*) (1 - q)R_A (\alpha - 2\beta q R_A A^*)
\]

(12)

and
\[(1-q)\delta u'\left(R_A A^* + R_B B^*\right)(R_A - R_B) - q\delta v'(R_B B^*)R_B
\]
\[= \kappa X'(z^*)\left[\alpha \left(R_B - (1-q) R_A\right) + 2\beta q \left(1-q\right) R_A^2 A^*\right]. \quad (13)\]

**Proof** Since the second-order condition holds under Lemma 1, the first-order conditions are the necessary and sufficient conditions for internal solutions. Moreover, from Equation (5), \( L = 0 \) could be written as
\[u'(w - A^* - B^*) = (1-q)\delta u'\left(R_A A^* + R_B B^*\right)R_A
\]
\[+ \kappa X'(z^*) \left(R_B - (1-q) A^* \right) \left(R_A - (1-q) R_B\right)\frac{\alpha}{\beta q R_A A^*}.\]
Subtracting \( L = 0 \) by \( M = 0 \) yields
\[
(1-q)\delta u'\left(R_A A^* + R_B B^*\right)(R_A - R_B) - q\delta v'(R_B B^*)R_B
\]
\[= \kappa X'(z^*)\left[\alpha \left(R_B - (1-q) R_A\right) + 2\beta q \left(1-q\right) R_A^2 A^*\right].
\]
where \( z^* = -\alpha \left((1-q) R_A A^* + R_B B^*\right) + \beta q \left(1-q\right) R_A^2 A^* \).

Q.E.D.

Proposition 1 shows the conditions for partial annuitization when the individual’s preferences exhibit anticipatory anxiety. It is obvious that Equation (13) will never hold when there is neither a bequest motive nor anticipatory anxiety as predicted by Yaari (1965). Moreover, in the absence of a bequest motive, if the individual’s preferences exhibit anticipatory anxiety, then the conditions in Proposition 1 may still hold and further ensure that partial annuitization is optimal. The following Corollary illustrates this case:

**Corollary 1** Suppose the conditions in Lemma 1 hold. In the absence of a bequest motive, the anxiety-averse individual will partially annuitize his/her wealth if and only if Equation (12) holds and
\[(1-q)\delta u^\prime\left(R_A^* - R_B^*\right)(R_A - R_B)\]
\[= \kappa X'(z')\left[\alpha \left(R_B - (1-q)R_A\right) + 2\beta q \left(1-q\right) R_A^2 A^*\right]. \quad (14)\]

This Corollary could be obtained directly from Proposition 1. Thus, the proof is omitted. In complementing the literature, Corollary 1 indicates that, even without a bequest motive, partial annuitization may be optimal when the individual is anxiety averse. In other words, anticipatory anxiety is one of the rationales for the annuity puzzle. The next Corollary shows the conditions for the extreme case where individuals with anticipatory anxiety invest in bonds but not in annuities:

**Corollary 2** Suppose the conditions in Lemma 1 hold. The anxiety-averse individual will not invest in an annuity if

\[(1-q)\delta u^\prime\left(R_B^*\right)(R_A - R_B) < q\delta v^\prime\left(R_B^*\right) + \kappa X'(\alpha R_B^* - (1-q)R_A^*). \quad (15)\]

**Proof** If the following conditions hold, then we will have \(A^* = 0\) and \(B^* > 0\) in equilibrium:

\[L_{A=0} = -u'(w-B) + (1-q)\delta u'(R_B)R_A + \kappa X'(-\alpha R_B)\left(1-q\right)R_A\alpha < 0 \quad (16)\]

and

\[M_{A=0} = -u'(w-B) + (1-q)\delta u'(R_B)R_B + q\delta v'(R_B)R_B + \kappa X'(\alpha R_B^* - \alpha R_B)\alpha R_B = 0. \quad (17)\]

Substituting Equation (17) into Equation (16) yields

\[-(1-q)\delta u'(R_B)R_B - q\delta v'(R_B)R_B - \kappa X'(\alpha R_B^* - \alpha R_B)\alpha R_B^* + (1-q)\delta u'(R_B^*)R_A + \kappa X'(\alpha R_B^*)\left(1-q\right)R_A\alpha < 0.\]

Rearranging the above equation yields
\[(1 - q)\delta u'(R_B B) (R_A - R_B) < q \delta v'(R_B B) R_B + \kappa X'(-\alpha R_B B^*) \alpha (R_B - (1 - q) R_A).\]

Q.E.D.

Corollary 2 shows the condition for \( A^* = 0 \) and \( B^* > 0 \). In the case where \( v' = 0 \) and \( R_A - (1 - q) R_B > 0 \), Corollary 2 indicates that if the anxiety coefficient \( \kappa \) is large enough, then the individual with anticipatory anxiety will only invest in bonds and will not invest in annuities at all. However, if the annuity is actuarially fairly priced, i.e.,

\[ R_A = \frac{R_B}{1 - q}, \]

then Corollary 2 predicts that \( A^* = 0 \) and \( B^* > 0 \) if the bequest motive is strong enough such that

\[ u'(R_B B) < v'(R_B B).\]

To have a better understanding of the effect of anxiety on annuitization, we further discuss the comparative statics in the following. We will focus on the effect of the change in \( \kappa \), \( \alpha \) and \( \beta \) on the optimal internal solution \( A^* \).

**Proposition 2** The demand for annuity \( A^* \) decreases as \( \kappa \) increases if

\[ \frac{\alpha}{\beta} < 2q R_A A^*. \]  

**Proof.** Since the second-order conditions hold, we can apply Cramer’s Rule and obtain

\[ \text{sign} \left\{ \frac{dA}{d\kappa} \right\} = \text{sign} \left\{ - \frac{dL}{d\kappa} \frac{dL}{dB} - \frac{dL}{d\kappa} \frac{dL}{dM} \right\}. \]

From the first-order conditions (5) and (6), the following derivatives can be calculated:
\[
\frac{dL}{d\kappa} = X'(z)(1-q)R_A \left( \alpha - 2\beta q R_A A' \right) \tag{20}
\]

and

\[
\frac{dM}{d\kappa} = X'(z)\alpha R_b. \tag{21}
\]

Furthermore, from Equations (9) and (10), we have

\[
\text{sign} \left\{ -\frac{dL}{d\kappa} \frac{dB}{dL} - \frac{dM}{d\kappa} \frac{dB}{dM} \right\} = \text{sign} \left\{ -\frac{dL}{d\kappa} \frac{dM}{dB} + \frac{dM}{d\kappa} \frac{dL}{dB} \right\}
\]

\[
= \text{sign} \left\{ -(1-q)R_A \left( \alpha - 2\beta q R_A A' \right) \left[ u''(w - A' - B') + (1-q)\delta u''(R_A A' + R_b B')R_b + q\delta v''(R_b B')R_b^2 \right] 
+ \alpha R_b \left[ u''(w - A' - B') + (1-q)\delta u''(R_A A' + R_b B')R_A R_b \right] \right\}
\]

(22)

Since \( u'' < 0 \) and \( v'' < 0 \), the sign of the above equation will be negative if \( \alpha - 2\beta q R_A A' < 0 \), or \( \frac{\alpha}{\beta} < 2q R_A A' \).

Q.E.D.

The above Proposition shows that an increase in the degree of anxiety \( \kappa \) will decrease the individual’s demand for an annuity if \( \frac{\alpha}{\beta} < 2q R_A A' \). An increase in \( \kappa \) will also increase the disutility from anxiety. Therefore, whether or not the individual demands more in terms of an annuity under an increase in \( \kappa \) will depend on the effect of purchasing the annuity on anticipatory anxiety. Note that

\[
\frac{dX(z')}{dA} = -\alpha (1-q)R_A + 2\beta q (1-q) R_A^2 A'. \tag{23}
\]

If \( \frac{\alpha}{\beta} < 2q R_A A' \), then the sign of the above equation will be positive, which means

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This condition will hold when \( \beta \), \( R_A \) and \( q \) are large enough and \( \alpha \) is sufficiently small.
that an increase in the purchase of the annuity will increase the anticipatory anxiety. Purchasing an additional annuity has two effects on anxiety. On the one hand, it will increase the average consumption and further decrease the anticipatory anxiety by $\alpha (1 - q) R_A$. On the other hand, it will increase the riskiness of future consumption and therefore increase the anticipatory anxiety by $2 \beta q (1 - q) R_A^2 A'$. Thus, whether or not an additional annuity will ease the anxiety depends on the relative strength of the above two effects. If the first effect is dominated by the second effect ($\frac{\alpha}{\beta} < 2 q R_A A'$), then an increase in the purchase of the annuity will increase the anxiety. Therefore, in this case, an increase in the degree of anxiety $\kappa$ will decrease the demand for annuity in the optimum so that the disutility from anxiety will be reduced.

**Proposition 3** The demand for annuity $A'$ increases as $\alpha$ increases if

$$\frac{X'(z^*)}{X^*(z^*)((1 - q) R_A A^* + R_B B^*)} < \alpha < 2 \beta q R_A A^* + \frac{X'(z^*)}{X^*(z^*)((1 - q) R_A A^* + R_B B^*)}.$$  \hfill (24)

**Proof.** Please see Appendix B.

Proposition 3 demonstrates the sufficient conditions for a positive relationship between the optimal investment in the annuity and $\alpha$. Note that if $\alpha$ or $\beta$ approach zero, then Equation (24) will not hold.

**Proposition 4** The demand for annuity $A'$ decreases as $\beta$ increases if

$$\frac{\alpha}{\beta} < 2 q R_A A'.$$  \hfill (25)

**Proof.** Please see Appendix C.
Proposition 4 shows the sufficient condition for a negative relationship between $A'$ and $\beta$. This condition is the same as the condition for a negative relationship between $A'$ and $\kappa$ as shown in Proposition 2. In other words, if investing in an annuity will increase the anxiety as shown in Equation (23), then an increase in $\kappa$ or $\beta$ will decrease the optimal purchase of the annuity because the reduction in the investment in the annuity is able to reduce the disutility from the anxiety.

To obtain a better understanding, we perform a simulation in the following section to demonstrate how the representative individual will allocate his/her assets with respect to the changes in the coefficients in our anxiety-averse consumption model.

III. Simulation

In the previous section, our theoretical results have shown that anticipatory anxiety may be one of the reasons why people do not fully annuitize their wealth. However, we do not answer the question as to why people invest in annuities at such a low level. In this section, we would like to use simulation to demonstrate that the investment in the annuity is sensitive to the degree of anxiety and that individuals will invest in annuities at a low level under a reasonable setting where there is anxiety.

Instead of the life-time models that have been widely used in Brown (2001), Davidoff, Brown and Diamond (2005), and Babbel and Merrill (2006), we conduct our simulation under the two-period model as in the previous section. In addition, to enrich our analysis, we assume that risky assets may be available while constructing an investment portfolio. Let $\bar{R}_s$ denote the gross return on the risky asset in the second period, where $\bar{R}_s$ is a random variable with expected return $\mu_s$ and
variance $\sigma_s^2$. We further assume that $\bar{R}_s = e^{\bar{r}_s}$, where $\bar{r}_s$ is normally distributed with mean $E[\bar{r}_s]$ and variance $\Sigma^2$.

Under the short selling constraint, the optimization problem faced by the anxiety-averse individual is:

$$\max_{A,B,S} U(C) = u(w - A - B - S)$$

$$+ \delta \left\{ (1-q)E \left[ u(R_A A + R_B B + \bar{R}_s S) \right] + qE \left[ v(R_B B + \bar{R}_s S) \right] \right\}$$

$$- \kappa X \left\{ -\alpha ((1-q)R_A A + R_B B + \mu_S S) + \beta \left( q(1-q)R_A^2 A^2 + \sigma_s^2 S^2 \right) \right\}$$

(26)

The form of the utility function is set as follows:

$$u(C) = \frac{C^{1-\rho}}{1-\rho},$$

which is known to exhibit constant relative risk aversion with $\rho$. When $\rho = 1$, the function simplifies to the case of log utility. These types of utility functions are commonly studied in the insurance economic literature. In this study, we examine the utility functions with constant relative risk aversion ranging from 2 to 6. For illustration purposes, we assume that the utility in relation to consumption and that for bequests both exhibit the same risk aversion level. Let $c$ denote the coefficient for the utility of a bequest where

$$c = \frac{v(C)}{u(C)}.$$

By setting the coefficient $c$ as less than unity, the effect of the bequest will not be amplified during the individual’s decision-making process. In the simulation, $c$ is set at 0.1. The anxiety function is assumed to take the following form:

$$X(z) = e^z - 1.$$  

(27)

Table 1 summarizes our choice of parameter values for the base case and ranges tested. Following Babbel and Merrill (2006), the net interest rate for bonds is set at 2%. The rate of time preference is set at 2% for the base case as well. The mortality
rate is set at 0.02, which is approximately the mortality rate of a person at the time of retirement, mainly in the age range of 65 to 70. With a given anxiety function, the parameter $\kappa$ measures the importance of the attribute of the disutility relative to the traditional utility. To set the value of $\kappa$ in the base case, we refer to our simulation results in Figure 1 which illustrates the optimal annuitization level for the individual with $\rho = 3$. As shown in the figure, the optimal annuitization level is about 20% when $\kappa$ is around 0.2. Thus, we set $\kappa = 0.2$ in the base case and test the effect of the parameter ranging from 0 to 1 in the simulation process. When $\kappa = 0$, the investor exhibits traditional risk averse utility.

Table 2 summarizes the optimal annuitization level under different settings. Case 1 is the standard Yaari model of full annuitization. Case 5 shows that when there is no bequest motive nor feeling of anxiety, the individual will still invest over 75% in the annuity even when the equity asset exists. This result is similar to that of Babbel and Merrill (2007). By comparing Cases 1 and 2, or Cases 5 and 6, we see that the bequest motive does decrease the optimal level of annuitization. Consistent with our theoretical prediction, when there is anticipatory anxiety, the individual will invest less in the annuity. Even when there is no bequest motive, Cases 3 and 7 show that the anxiety-averse individual is willing to invest less than 10% of the assets in the annuity after consumption.

The relationship between the degree of risk aversion and the optimal annuitization level in the absence of anxiety is demonstrated in Figure 2(a). The $x$-axis in Figure 2(a) represents the individual’s degree of relative risk-aversion, while the $y$-axis represents the summation of $A/(A + B + S)$, $B/(A + B + S)$ and $S/(A + B + S)$. Figure 2(a) demonstrates that when there is a bequest motive, a risk-averse individual with no anticipatory anxiety may invest about 32% to 52% of his assets in the annuity after consumption. These results are similar to those of past
studies but higher than the corresponding percentages that we would observe in the real world. On the other hand, Figure 2(b) presents the relationship between degree of risk aversion of the anxiety-averse individual and its optimal portfolio in the absence of bequest motives. Consistent with our conjecture from Corollary 1, Figure 2(b) demonstrates that when there is no bequest motive, an anxiety-averse individual may invest less than 65% of assets in annuity and the percentage may even get lower than 10% for higher risk-aversion individuals.

Our Proposition 2 depicts the condition for the decrease in the purchase of the annuity when the degree of anxiety increases. The relationship between the optimal annuitization level and the degree of anxiety is illustrated in Figure 3. In Figure 3, the $x$-axis represents $\kappa$ and the $y$-axis represents the percentage of wealth annuitized after consumption, i.e., $A/(A+B+S)$. Figure 3 shows that an increase in $\kappa$ will decrease the individual’s demand for the annuity and the decrease will become more severe as the individual’s degree of relative risk-aversion increases. The results coincide with our theoretical conclusion in the previous section. The simulations show that, for higher values of $\kappa$, the optimal annuitization level is less than 20% for less-risk-averse individuals and may be even less than 1% for highly-risk-averse individuals.

Our Proposition 2 concludes that when $\beta$ is sufficiently large relative to $\alpha$, the effect on anxiety through the variance will dominate. In this case, anxiety will reduce the individual’s demand for the annuity. As shown in Figure 4, when the anxiety of the individual comes from the risk related to the return on his/her assets, s/he will purchase more bonds when $\kappa$ increases. On the other hand, as in Figure 5, the percentage of assets annuitized will remain merely unchanged as $\kappa$ increases when $\beta$ is small.

Figure 6 demonstrates that, when $\alpha$ increases, the degree of anxiety decreases
and the individual will buy more annuities and risk assets. On the other hand, Figure 7 shows that when $\beta$ increases, the degree of anxiety that the individual feels regarding the variance in both the annuity and the risky asset will increase, and thus s/he will allocate more of his/her assets in bonds.

### IV. Conclusion

Although many studies have successfully shown why individuals partially annuitize their income, only a few papers can explain why individuals annuitize their income at a low level. In this paper, we provide another rationale, namely, anticipatory anxiety, to explain why individuals annuitize their income at a very low level.

We incorporate the anxiety in the traditional two-period consumption model to show how the demand for the annuity is sensitive to the degree of anxiety. We also find the necessary and sufficient conditions for partial annuitization to be optimal. When an annuity increases the level of anxiety, our theoretical model will predict a negative relationship between the degree of anticipatory anxiety and the optimal demand for annuities. By constructing a simulation, we demonstrate that, given reasonable parameters, anxiety can explain why individuals annuitize their income at a relatively low level.

Our paper contributes to the literature by pointing out that perspectives regarding individual behavior are important to annuitization. Therefore, understanding whether government policy could affect the individual’s annuitization decision by correcting such behavioral perspectives and further enhancing social welfare could yield some useful results in future research.
References


Table 1  Parameter values for the base case and ranges tested

<table>
<thead>
<tr>
<th>Variables</th>
<th>Base Case</th>
<th>Ranges Tested</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net interest on bonds, $r_B = \ln(R_B)$</td>
<td>0.02</td>
<td></td>
</tr>
<tr>
<td>Rate of time preference, $\delta$</td>
<td>0.02</td>
<td></td>
</tr>
<tr>
<td>Equity risk premium, $E[\tilde{r}_s] - r_B$</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td>Equity volatility, $\Sigma$</td>
<td>0.20</td>
<td></td>
</tr>
<tr>
<td>Coefficient for the utility of bequests, $c$</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td>Mortality rate, $q$</td>
<td>0.02</td>
<td></td>
</tr>
<tr>
<td>Coefficient of risk aversion, $\rho$</td>
<td>3.00</td>
<td>2 to 6</td>
</tr>
<tr>
<td>Parameter for anxiety on the average consumption, $\alpha$</td>
<td>1.00</td>
<td>0 to 2</td>
</tr>
<tr>
<td>Parameter for anxiety on the variance of consumption, $\beta$</td>
<td>2.00</td>
<td>0 to 5</td>
</tr>
<tr>
<td>Parameter for the degree of anxiety, $\kappa$</td>
<td>0.20</td>
<td>0 to 1</td>
</tr>
</tbody>
</table>
Table 2  Optimal annuitization level under different settings

<table>
<thead>
<tr>
<th>Case</th>
<th>Coefficient for the utility of bequests, $c$</th>
<th>Parameter for anxiety, $\kappa$</th>
<th>Risky assets included</th>
<th>Optimal Annuitization Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>N</td>
<td>100%</td>
</tr>
<tr>
<td>2</td>
<td>0.1</td>
<td>0</td>
<td>N</td>
<td>53.1%</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0.2</td>
<td>N</td>
<td>26.8%</td>
</tr>
<tr>
<td>4</td>
<td>0.1</td>
<td>0.2</td>
<td>N</td>
<td>21.8%</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
<td>Y</td>
<td>75.3%</td>
</tr>
<tr>
<td>6</td>
<td>0.1</td>
<td>0</td>
<td>Y</td>
<td>52.0%</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>0.2</td>
<td>Y</td>
<td>25.4%</td>
</tr>
<tr>
<td>8</td>
<td>0.1</td>
<td>0.2</td>
<td>Y</td>
<td>20.7%</td>
</tr>
</tbody>
</table>

Note: In all cases, we set $\rho = 3$, $\alpha = 1$ and $\beta = 2$
Figure 1 Optimal annuitization for varying $\kappa$ in the base case
Figure 2 Initial asset allocation for varying risk aversion in the base case

(a) ($\kappa = 0$)

(b) ($c = 0$)
Figure 3 Optimal annuitization for varying $\kappa$ in the base case
Figure 4 Initial asset allocation for varying $\kappa$ with $\alpha = 1$, $\beta = 2$ and $\rho = 3$
Figure 5  Initial asset allocation for varying $\kappa$ with $\alpha = 1$, $\beta = 0.1$ and $\rho = 3$
Figure 6 Initial asset allocation for varying $\alpha$

$(\kappa=0.2, \beta=2, \rho=3)$
Figure 7 Initial asset allocation for varying $\beta$

$(\kappa = 0.2, \alpha = 1, \rho = 3)$
Appendix A  Proof of Lemma 1

From Equations (7) to (10), we know that
\[
\begin{vmatrix}
\frac{dL}{dA} & \frac{dL}{dB} \\
\frac{dM}{dA} & \frac{dM}{dB}
\end{vmatrix} = \frac{dL \, dM}{dA \, dB} - \frac{dL \, dM}{dB \, dA}
= u''(w - A - B)(1 - q)\delta u''(R_xA + R_yB)(R_x - R_y)^2 \\
+ \left[ u''(w - A - B) + (1 - q)\delta u''(R_xA + R_yB)R_x^2 \right] q\delta v''(R_yB)R_y^2 \\
- \kappa X''(z)(\alpha R_y)^2 \left\{ u''(w - A - B) + (1 - q)\delta u''(R_xA + R_yB)R_y^2 \right\} \\
- \kappa X''(z) \left[ (1 - q) R_x (\alpha - 2\beta qR_xA) \right]^{-\frac{1}{2}} \left\{ u''(w - A - B) + (1 - q)\delta u''(R_xA + R_yB)R_y^2 \right\} \left\{ +q\delta v''(R_yB)R_y^2 - \kappa X''(z)(\alpha R_y)^2 \right\} \\
- \kappa X''(z)2\beta q(1 - q)R_x^2 \left\{ u''(w - A - B) + (1 - q)\delta u''(R_xA + R_yB)R_y^2 \right\} \left\{ q\delta v''(R_yB)R_y^2 - \kappa X''(z)(\alpha R_y)^2 \right\} \\
+ \kappa X''(z)(\alpha - 2\beta qR_xA)\alpha (1 - q) R_xR_y \left\{ 2u''(w - A - B) \right\} \left\{ +2(1 - q)\delta u''(R_xA + R_yB)R_xR_y \right\} \left\{ -\kappa X''(z)\alpha (1 - q) R_xR_y \left( \alpha - 2\beta qR_xA \right) \right\}.
\]
\(\text{Equation (A1)}\)

From the assumptions of \(u'' < 0\), \(v'' < 0\) and \(X'' \geq 0\), we know that every term in Equation (A1) is non-negative except for the last term. If \(X'' = 0\), then Equation (A1) will be positive, and, therefore, the second-order condition will hold. If \(X'' > 0\), then Equation (A1) will be positive when the last term in Equation (A1) is also positive, i.e.,
\[
\kappa X''(z)(\alpha - 2\beta qR_xA)\alpha (1 - q) R_xR_y \left\{ 2u''(w - A - B) + 2(1 - q)\delta u''(R_xA + R_yB)R_xR_y \right\} \left\{ -\kappa X''(z)\alpha (1 - q) R_xR_y \left( \alpha - 2\beta qR_xA \right) \right\} > 0.
\]
\(\text{Equation (A2)}\)

Since \(\kappa\), \(X''\), \(\alpha\), \(1 - q\), \(R_x\) and \(R_y\) are all positive, Equation (A2) is equivalent to
\[ 2(\alpha - 2\beta q R_{x,A})\left[u^*(w - A - B) + (1 - q)\delta u^*(R_{x,A} + R_{y,B})R_{x,R_y}\right] > (\alpha - 2\beta q R_{x,A})^2 \kappa X^*(z)\alpha (1 - q)R_{x,R_y}. \]  

(A3)

If \( \alpha - 2\beta q R_{x,A} > 0 \) for all \( A \), then Equation (A3) could be written as

\[
\alpha - 2\beta q R_{x,A} < \frac{2\left[u^*(w - A - B) + (1 - q)\delta u^*(R_{x,A} + R_{y,B})R_{x,R_y}\right]}{\kappa X^*(z)\alpha (1 - q)R_{x,R_y}} < 0,
\]

(A4)

which contradicts the assumption of \( \alpha - 2\beta q R_{x,A} > 0 \). Therefore, \( \alpha - 2\beta q R_{x,A} > 0 \) for all \( A \) cannot stand for Equation (A3). On the other hand, if \( \alpha - 2\beta q R_{x,A} < 0 \) for all \( A \), then Equation (A3) could be written as

\[
0 > \alpha - 2\beta q R_{x,A} > \frac{2\left[u^*(w - A - B) + (1 - q)\delta u^*(R_{x,A} + R_{y,B})R_{x,R_y}\right]}{\kappa X^*(z)\alpha (1 - q)R_{x,R_y}}.
\]

(A5)

In other words, \( X^* > 0 \) and Equation (A5) are the sufficient conditions for the second-order condition of the individual’s optimization problem.

Q.E.D.
Appendix B  Proof of Proposition 3

Since the second-order conditions hold, we can apply Cramer’s rule and obtain

\[
\text{sign} \left\{ \frac{dA}{d\alpha} \right\} = \text{sign} \left\{ -\frac{dL}{d\alpha} \frac{dL}{dB} - \frac{dL}{dM} \frac{dM}{dB} \right\} = \text{sign} \left\{ -\frac{dL}{d\alpha} \frac{dM}{dB} + \frac{dM}{d\alpha} \frac{dL}{dB} \right\},
\]

where

\[
\frac{dL}{d\alpha} = \kappa X'(z^*) (1-q) R_A - \kappa X''(z^*) ((1-q) R_A A^* + R_B B^*) (1-q) R_A (\alpha - 2\beta q R_A A^*)
\]

(B1)

and

\[
\frac{dM}{d\alpha} = \kappa X'(z^*) R_B - \kappa X''(z^*) ((1-q) R_A A^* + R_B B^*) \alpha R_B.
\]

(B2)

From Equation (10), we have \(\frac{dM}{dB} < 0\). If \(\alpha - 2\beta q R_A A^* > 0\), then \(\frac{dL}{dB} < 0\) from Equation (9). If \(\frac{dL}{d\alpha} > 0\) and \(\frac{dM}{d\alpha} < 0\) under the assumption \(\alpha - 2\beta q R_A A^* > 0\), then \(\frac{dA^*}{d\alpha}\) will be positive. Note that \(\frac{dL}{d\alpha} > 0\) could be written as

\[
\kappa X'(z^*) (1-q) R_A > \kappa X''(z^*) ((1-q) R_A A^* + R_B B^*) (1-q) R_A (\alpha - 2\beta q R_A A^*).
\]

Or,

\[
\alpha - 2\beta q R_A A^* < \frac{X'(z^*)}{X''(z^*) ((1-q) R_A A^* + R_B B^*)}.
\]

(B3)

\(\frac{dM}{d\alpha} < 0\) could be written as

\[
\kappa X'(z^*) R_B < \kappa X''(z^*) ((1-q) R_A A^* + R_B B^*) \alpha R_B.
\]

Or,

\[
\frac{X'(z^*)}{X''(z^*) ((1-q) R_A A^* + R_B B^*)} < \alpha.
\]

(B4)

To sum up, from the above discussion and Equations (B3) and (B4), we know that
\[ \frac{dA^*}{d\alpha} > 0 \quad \text{if} \quad 0 < \alpha - 2 \beta q R_x A^* < \frac{X'(z^*)}{X''(z^*) \left( (1 - q) R_x A^* + R_y B^* \right)} < \alpha. \]

Rearranging the above equation yields

\[ \frac{X'(z^*)}{X''(z^*) \left( (1 - q) R_x A^* + R_y B^* \right)} < \alpha < 2 \beta q R_x A^* + \frac{X'(z^*)}{X''(z^*) \left( (1 - q) R_x A^* + R_y B^* \right)}. \]

Q.E.D.
Appendix C  Proof of Proposition 4

Similar to the proof of Proposition 3, based on the second-order condition, Cramer’s rule ensures that

\[
\text{sign}\left\{ \frac{dA}{d\beta} \right\} = \text{sign}\left\{ \begin{vmatrix} \frac{dL}{d\beta} & \frac{dL}{dB} \\ \frac{dM}{d\beta} & \frac{dM}{dB} \end{vmatrix} \right\} = \text{sign}\left\{ -\frac{dL}{d\beta} \frac{dM}{dB} + \frac{dM}{d\beta} \frac{dL}{dB} \right\},
\]

where

\[
\frac{dL}{d\beta} = -2\kappa X'(z^*) (1-q) q R_A^2 A^* + \kappa X''(z^*) (1-q) R_A \left( \alpha - 2\beta q R_A A^* \right) q (1-q) R_A^2 A^{*2},
\]

(C1)

and

\[
\frac{dM}{d\beta} = \kappa X''(z^*) \alpha R_B q \left( 1-q \right) R_A^2 A^{*2}.
\]

(C2)

Thus, we have

\[
-\frac{dL}{d\beta} \frac{dM}{dB} + \frac{dM}{d\beta} \frac{dL}{dB} = 2\kappa X'(z^*) (1-q) q R_A^2 A^* \left[ u''(w-A-B) + (1-q) \delta u''(R_A A + R_B B) R_B^2 \right] + q \delta v''(R_B B) R_B^2 - \kappa X''(z) \left( \alpha R_B \right)^2
\]

\[
+ \kappa X''(z^*) \alpha R_B q \left( 1-q \right) R_A^2 A^{*2} \left[ u''(w-A-B) + (1-q) \delta u''(R_A A + R_B B) R_A R_B \right]
\]

\[
- \kappa X''(z^*) (1-q)^2 \left( \alpha - 2\beta q R_A A^* \right) q R_A^3 A^{*2} \left[ u''(w-A-B) + q \delta v''(R_B B) R_B^2 \right].
\]

The first two terms in the above equation are negative from the assumption of \( u'' < 0, \ v'' < 0, \ X' > 0 \) and \( X'' \geq 0 \). Thus, if \( \alpha - 2\beta q R_A A^* < 0 \), or \( \frac{\alpha}{\beta} < 2q R_A A^* \), then the last term in the above equation will also be negative, and will result in a negative relationship between the investment in the annuity and \( \beta \).

Q.E.D.